# Engineering

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http://denisemeeks.com/science/notebooks/notebook_engineering.pdf

## Engineering: Engineering Method

- Define the problem. Why is this a problem?
- Do background research. Have others investigated this if so, what did they discover? Does this information help you?
- Specify requirements. What are the time, resource, cost, and technical requirements?
- Brainstorm possible solutions. Communicate, cooperate, and collaborate with colleagues.
- Choose the best solution. Why do you think the solution you chose will work?
- Develop the project. Gather the required resources.
- Build a prototype. What works? What doesn't work?
- Test and redesign. Make the required changes to your design.

## Engineering: Systems

system: group of interacting, interrelated, or interdependent elements forming a complex whole
closed system: no matter enters or leaves the system
open system: energy and matter can enter or leave the system
positive feedback: mechanism that enhances or drives a change
negative feedback: mechanism that works to maintain existing conditions

## Engineering: Engineering Notation

engineering notation: $m \; x10^n$, where $n$ is an integer divisible by 3, meaning that $1 \le m < 1{,}000$

## Engineering: Polar Coordinates

particle position $\vec{r} = r\vec{u}_r$

particle instantaneous velocity $\vec{v} = v_r\vec{u}_r + v_\theta\vec{u}_\theta = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta$

velocity magnitude $v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

particle instantaneous acceleration

$$\vec{a} = \dot{\vec{v}} = a_r\vec{u}_r + a_\theta\vec{u}_\theta = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta$$

$$= \ddot{r}\vec{u}_r + \dot{r}\dot{\vec{u}}_r + \dot{r}\dot{\theta}\vec{u}_\theta + r\ddot{\theta}\vec{u}_\theta + r\dot{\theta}\dot{\vec{u}}_\theta$$

acceleration magnitude $a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$

## Engineering: Cylindrical Coordinates

particle position $\vec{r}_p = r\vec{u}_r + z\vec{u}_z$

particle instantaneous velocity

$$\vec{v} = v_r\vec{u}_r + v_\theta\vec{u}_\theta + v\vec{u}_z = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta + \dot{z}\vec{u}_z$$

velocity magnitude $v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

particle instantaneous acceleration

$$\vec{a} = \dot{\vec{v}} = a_r\vec{u}_r + a_\theta\vec{u}_\theta = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta + \ddot{z}u_z$$

acceleration magnitude $a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$

## Engineering: Steady Fluid Streams

constant mass:

$$\sum F_x = \frac{dm}{dt}(\vec{v}_{Bx} - \vec{v}_{Ax}) \qquad \sum F_y = \frac{dm}{dt}(\vec{v}_{By} - \vec{v}_{Ay})$$

$$\frac{dm}{dt} = \rho_A v_A A_A = \rho_B v_B A_B = \rho_A Q_A = \rho_B Q_B$$

mass loss:

$$\sum F_s = m\frac{dv}{dt} - (v + v_e)\frac{dm_e}{dt}$$

$v_e$ = velocity of expelled mass $\qquad \frac{dm_e}{dt}$ = rate of mass loss

## Engineering: Vibrations and Simple Harmonic Motion

$$-kx = m\ddot{x} = m\dot{v} = ma$$

standard form oscillating spring $\ddot{x} + p^2 x = 0 \qquad p = \sqrt{\frac{k}{m}}$

$$x = A\sin pt + B\cos pt = \frac{v_1}{p}\sin pt + x_1 \cos pt$$

$$v = \dot{x} = Ap\cos pt + Bp\sin pt \qquad A = C\cos\phi$$

$$a = \ddot{x} = -Ap^2 \sin pt - Bp^2 \cos pt \qquad B = C\sin\phi$$

$$x = C\cos\phi \sin pt + C\sin\phi \cos pt$$

suspended block: $\ddot{y} + p^2 y = 0$

## Engineering: Undamped Forced Vibrations

$$F_{\max}\sin\omega t - kx = m\ddot{x} \qquad \ddot{x} + \frac{k}{m}x = \frac{F_{\max}}{m}\sin\omega t$$

complementary solution $x_c = A\sin pt + B\cos pt$

particular solution $x_p = C\sin\omega t \qquad \ddot{x}_p = -C\omega^2 \sin\omega t$

$F_{\max}$ = maximum force $\qquad k$ = spring constant

$m$ = mass $\qquad \omega$ = forcing frequency

$$-C\omega^2 \sin\omega t + \frac{k}{m}(C\sin\omega t) = \frac{F_{\max}}{m}\sin\omega t \qquad x_p = \frac{F_{\max}/k}{1 - \left(\frac{\omega}{p}\right)^2}$$

## Engineering: Mechanical Properties of Materials (1)

brittleness: ability of a material to break or shatter without significant deformation when under stress; opposite of plasticity

bulk modulus: ratio of pressure to volumetric compression (GPa)

$$K = -V\frac{dP}{dV} \qquad P = \text{pressure} \qquad V = \text{volume}$$

coefficient of friction: dimensionless scalar value $\mu$ which describes ratio of the force of friction between two bodies and force pressing them together

coefficient of restitution: $e$, measure of "restitution" of a collision between two objects; how much of the kinetic energy remains for rebounding vs. how much is lost as heat or work of deformation

$$e = \frac{\text{relative velocity after collision}}{\text{relative velocity before collision}}$$

## Engineering: Mechanical Properties of Materials (2)

compressive strength: maximum stress a material can withstand before compressive failure (MPa); corresponds to the point on the engineering stress strain curve $(\varepsilon_e^*, \sigma_e^*)$ defined by

$$\varepsilon_e^* = \frac{l^* - l}{l_0} \qquad \sigma_e^* = \frac{F^*}{A_0}$$

$l^*$ = length just before crushing

$l_0$ = original length $\qquad F^*$ = load just before crushing $\qquad A_0$ = original area

## Engineering: Mechanical Properties of Materials (3)

creep: slow and gradual deformation of an object with respect to time

$$\frac{d\varepsilon}{dt} = \frac{C\sigma^m}{d^b}e^{-Q/kT}$$

$\varepsilon$ = creep strain

$C$ = constant dependent on the material and creep mechanism

$m$ = exponent depending on creep mechanism

$b$ = exponent dependent on the creep mechanism

$Q$ = activation energy of the creep mechanism

$\sigma$ = applied stress $\qquad d$ = grain size of the material

$k$ = Boltzmann's constant $\qquad T$ = absolute temperature

## Engineering: Mechanical Properties of Materials (4)

ductility: ability of a material to deform under tensile stress

durability: Ability to withstand wear, pressure, or damage; hard-wearing

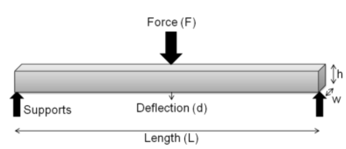
elasticity: Ability of a body to resist a distorting influence or stress and to return to its original size and shape when the stress is removed

$\sigma = E\varepsilon \qquad \sigma$ = stress $\qquad E$ = constant $\qquad \varepsilon$ = strain

fatigue limit: maximum stress a material can withstand under repeated loading (MPa)

flexural modulus: intensive property that is computed as the ratio of stress to strain in flexural deformation

$$E_{bend} = \frac{L^3 F}{4wh^3 d}$$

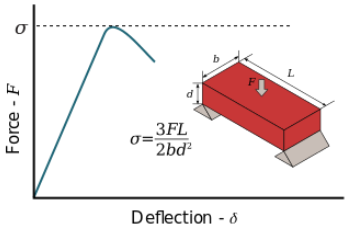



(Image source: https://en.wikipedia.org/wiki/Flexural_modulus#/media/File:Flexural_modulus_measurement.png, Adjwilley, CC SA-BY 4.0)

## Engineering: Mechanical Properties of Materials (5)

flexural strength: material property, defined as the stress in a material just before it yields in a flexure test

$$\sigma = \frac{3FL}{2bd^2}$$




fracture toughness: ability of a material containing a crack to resist fracture (MPa $m^{1/2}$)

hardness: measure of how resistant solid matter is to various kinds of permanent shape change when a compressive force is applied

(Image source: https://en.wikipedia.org/wiki/Flexural_strength#/media/File:Flexural_strength.svg, Nicoguaro, CC BY 4.0)

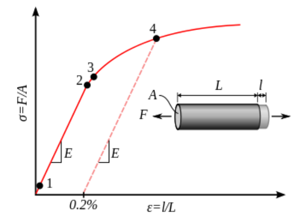
## Engineering: Mechanical Properties of Materials (6)

malleability: material's ability to deform under compressive stress

plasticity: describes the deformation of a solid material undergoing non-reversible changes of shape in response to applied forces

Poisson's ratio: length change $\nu \approx \frac{\Delta L'}{\Delta L}$

volume change $\frac{\Delta V}{V} \approx (1-2\nu)\frac{\Delta L}{L}$




(Image sources https://en.wikipedia.org/wiki/Plasticity_(physics)#/media/File:Metal_yield.svg, Sigmund, public domain)

1 = true elastic limit; 2 = proportionality limit, 3 = elastic limit, 4 = offset yield strength

## Engineering: Mechanical Properties of Materials (7)

resilience: the ability of a material to absorb energy when it is deformed elastically, and release that energy upon unloading

modulus of resilience $U_r = \frac{\sigma_y^2}{2E} = \frac{\sigma_y * \varepsilon_y}{2}$ $\sigma_y$ = yield strength

$\varepsilon_y$ = yield strain $E$ = Young's modulus

shear modulus: ratio of shear stress to shear strain

$G = \frac{\tau_{xy}}{\gamma_{xy}} = \frac{Fl}{A\Delta x}$ $\tau_{xy} = F/A$ = shear stress $\gamma_{xy} = \Delta x / l$ = shear strain

$F$ = force $l$ = initial length $A$ = area $\Delta x$ = transverse displacement

## Engineering: Mechanical Properties of Materials (8)

shear strength: strength of a material or component against the type of yield or structural failure where the material or component fails in shear

$\tau = \frac{\sigma_1 - \sigma_3}{2}$ $\sigma_1$ = major principle stress $\sigma_3$ = minor principle stress

specific modulus: consisting of the elastic modulus per mass density of a material

specific strength: material's strength divided by its density

specific weight: weight per unit volume

stiffness: measure of the resistance offered by an elastic body to deformation

$k = \frac{F}{\delta}$ $k$ = stiffness $F$ = force applied on the body

$\delta$ = displacement produced by force along same degree of freedom

surface roughness: quantified by the deviations in the direction of the normal vector of a real surface from its ideal form

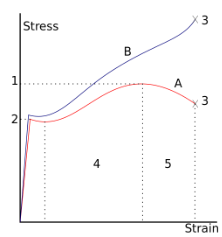
## Engineering: Mechanical Properties of Materials (9)

tensile strength: capacity of a material or structure to withstand loads tending to elongate, as opposed to compressive strength, which withstands loads tending to reduce size

toughness: ability of a material to absorb energy and plastically deform without fracturing

$\frac{\text{energy}}{\text{volume}} = \int_0^{\varepsilon_f} \sigma d\varepsilon$ $\sigma$ = stress

$\sigma$ = strain $\varepsilon_f$ = strain at failure




1=ultimate strength, 2=yield strength, 3=rupture, 4=strain hardening region, 5=necking region, A=apparent stress, B=actual stress

(Image source: https://en.wikipedia.org/wiki/Ultimate_tensile_strength#/media/File:Stress_v_strain_A36_2.svg, David Richfield, CC BY-SA 3.0)

## Engineering: Mechanical Properties of Materials (10)

viscosity: measure of its resistance to gradual deformation by shear stress or tensile stress

$F = \mu A \frac{u}{y}$ $F$ = force $\mu$ = proportionality factor $A$ = plate area

$u$ = speed $y$ = separation

yield strength: stress at which a material begins to deform plastically

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3) \le \sigma_y^2$$

Young's modulus: measure of the stiffness of a solid material

$$E = \frac{\sigma(\varepsilon)}{\varepsilon} = \frac{FL_0}{A_0 \Delta L}$$

$F$ = force $L_0$ = initial length $A_0$ = initial area $\Delta L$ = change of length

## Engineering: Uniaxial Load Deformation

stress on cross section $\sigma = \frac{P}{A}$

$P$ = loading $A$ = cross-sectional area

$\varepsilon = \frac{\partial}{L}$ $E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\delta / L}$

$\partial$ = elastic longitudinal deformation

$L$ = length of member

## Engineering: Cylindrical Pressure Vessel

internal pressure tangential hoop stress $\sigma_t = P_i \frac{r_0^2 + r_i^2}{r_0^2 - r_i^2}$

external pressure tangential hoop stress $\sigma_t = -P_0 \frac{r_0^2 + r_i^2}{r_0^2 - r_i^2}$

$\sigma_r$ = radial stress

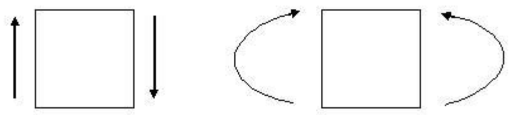
$P_i$ = internal pressure $P_0$ = external pressure

$r_i$ = inside radius $r_0$ = outside radius

for vessels with end caps axial stress $\sigma_a = P_i \frac{r_i^2}{r_0^2 - r_i^2}$

**Engineering: Shearing Force and Bending Moment Sign Conventions**

1. Bending moment is positive if it produces bending of the beam concave upward (compression in top fibers and tension in bottom fibers).
2. The shearing force is positive if the right portion of the beam tends to shear downward with respect to the left.



(Image source: https://en.wikipedia.org/wiki/Shear_and_moment_diagram#/media/File:Shear_and_Moment_Convention.jpg, Brett Mattas, public domain)

**Engineering: Fluid Drag Flow**

drag force on objects immersed in a large body of flowing fluid or objects moving through a stagnant fluid $F_D = \frac{C_D \rho V^2 A}{2}$

$C_D$ = drag coefficient $\rho$ = fluid density

$V$ = velocity (m/s) of the undisturbed fluid

$A$ = projected area ($m^2$) of blunt objects such as spheres, ellipsoids, disks, and plates, cylinders, ellipses, and air foils with axes perpendicular to the flow.

**Engineering: Airfoil Theory (1)**

lift force on an airfoil $F_L = \frac{C_L \rho V^2 A_P}{2}$

$C_L$ = lift coefficient $\rho$ = fluid density

$V$ = velocity (m/s) of undisturbed fluid

$A_P$ = projected area of the airfoil as seen from above

lift coefficient $C_L = 2\pi k_1 \sin(\alpha + \beta)$

$k_1$ = constant of proportionality

$\alpha$ = angle of attack (angle between airfoil chord and direction of flow)

β = negative of angle of attack for zero lift.

**Engineering: Airfoil Theory (2)**

drag coefficient $C_D \approx C_{D\infty} + \frac{C_L^2}{\pi AR}$

$C_{D\infty}$ = infinite span drag coefficient

$AR = \frac{b^2}{A_P} = \frac{A_P}{c^2}$

aerodynamic moment $M = \frac{C_M V^2 A_P c}{2}$

$C_M$ = moment coefficient

$A_P$ = plan area

$c$ = chord length

**Engineering: Reynolds Number**

Newtonian fluid Reynolds number $\text{Re} = \frac{VD\rho}{\mu} = \frac{VD}{v}$

$\rho$ = the mass density

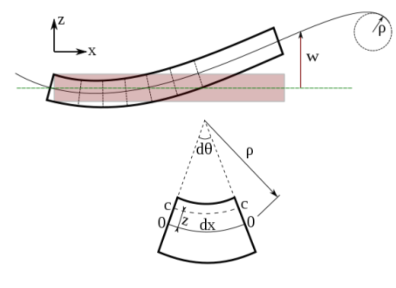
$D$ = pipe diameter or dimension of the fluid streamline

$\mu$ = dynamic viscosity

$v$ = kinematic viscosity

**Engineering: Static Beam Equation**

$$\frac{d^2}{dx^2}\left(EI \frac{d^2 w(x)}{dx^2}\right) = q$$




$E$ = elastic modulus

$I$ = second moment of area

$w(x)$ = equation describing the deflection of the beam in the $z$ direction at some position $x$

$q$ = force/area

(Image source: https://en.wikipedia.org/wiki/Euler%E2%80%93Bernoulli_beam_theory#/media/File:Euler-Bernoulli_beam_theory-2.svg, MIntz, CC BY-SA 3.0)

**Engineering: Sheer Stress in a Shaft and Polar Moment of Inertia**

sheer stress in a shaft $\tau = \frac{Tr}{J}$ (MPa, psi)

$T$ = twisting moment (Nmm, lb)

$r$ = distance from center to stressed surface in given position (mm, in)

$J$ = polar moment of inertia of an area ($mm^4$, $in^4$)

polar moment of inertia: a measure of a beam's ability to resist torsion; defined with respect to an axis perpendicular to the area considered; analogous to the "Area Moment of Inertia" which characterizes a beam's ability to resist bending required to predict deflection and stress in a beam; also called Second Moment of Area", "Area Moment of Inertia", "Polar Moment of Area" or "Second Area Moment"

**Engineering: Circular Shaft and Maximum Moment or Torque**

maximum moment in a circular shaft: $T_{max} = \frac{\tau_{max} R}{J}$ (Nmm, in lb)

$\tau_{max}$ = maximum shear stress (MPa, psi)

$R$ = radius of shaft (mm, in)

solid shaft $T_{max} = \frac{\pi}{16} \tau_{max} D^3$

hollow shaft $T_{max} = \frac{\pi}{16} \tau_{max} \frac{(D^4 - d^4)}{D}$

$D$ = outside diameter = $1.72(T_{max} / \tau_{max})^{1/3}$

$d$ = inside diameter

## Engineering: Tortional Deflection of a Shaft

angular shaft deflection $\theta = \frac{LT}{JG}$

$L$ = length of shaft (mm, in)
$G$ = modulus of rigidity (MPa, psi)

angular deflection of a torsion solid shaft $\theta = \frac{32LT}{G\pi D^4}$

angular deflection of a torsion hollow shaft $\theta = \frac{32LT}{G\pi (D^4 - d^4)}$

## Engineering: Specific Heat Capacities for Gases

| Substance | Mol wt | $c_P$ in kJ/kg K | $c_V$ in kJ/kg K | k |
|---|---|---|---|---|
| air | 29 | 1.00 | 0.718 | 1.40 |
| argon | 40 | 0.520 | 0.312 | 1.67 |
| butane | 58 | 1.72 | 1.57 | 1.09 |
| carbon dioxide | 44 | 0.846 | 0.657 | 1.29 |
| carbon monoxide | 28 | 1.04 | 0.744 | 1.40 |
| ethane | 30 | 1.77 | 1.49 | 1.18 |
| helium | 4 | 5.19 | 3.12 | 1.67 |
| hydrogen | 2 | 14.3 | 10.2 | 1.40 |
| methane | 16 | 2.25 | 1.74 | 1.30 |
| neon | 20 | 1.03 | 0.618 | 1.67 |
| nitrogen | 28 | 1.04 | 0.743 | 1.40 |
| octane vapor | 114 | 1.71 | 1.64 | 1.04 |
| oxygen | 32 | 0.918 | 0.658 | 1.40 |
| propane | 44 | 1.68 | 1.49 | 1.12 |
| steam | 18 | 1.87 | 1.41 | 1.33 |

## Engineering: Specific Heat Capacities for Liquids and Solids

| substance | $c_P$ kJ/kg K | density kg/m$^3$ |
|---|---|---|
| | liquids | |
| ammonia | 4.80 | 602 |
| mercury | 0.139 | 13,560 |
| water | 4.18 | 997 |
| | solids | |
| aluminum | 0.900 | 2,700 |
| copper | 0.386 | 8,900 |
| ice at 0 C | 2.11 | 917 |
| iron | 0.450 | 7,840 |
| lead | 0.128 | 11,310 |