I. Objectives

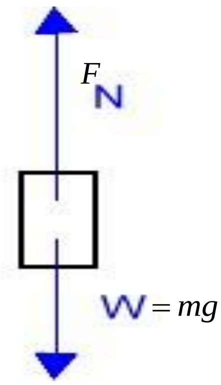
1. Investigate the concepts of work and energy.
2. Identify maximum and minimum kinetic and potential energy of a ball moving in a vertical circular path.

II. Introduction

Part 1 Vertical Motion

A suspended object moving with constant velocity experiences two forces:

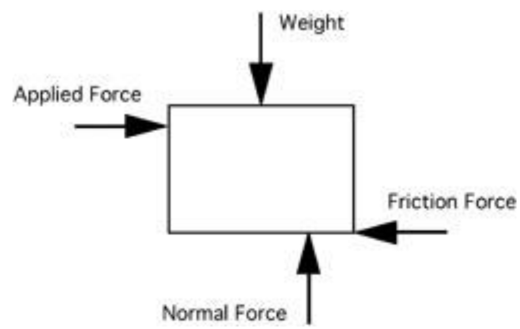
- the force preventing it from falling, in the upward direction,
- the force due to the acceleration of gravity, in the downward direction.



Part 2 Horizontal Motion

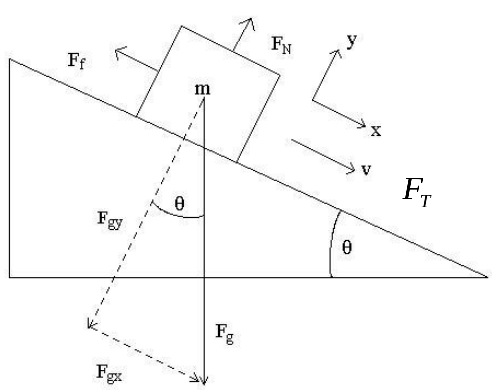
An object moving along a horizontal surface with constant velocity experiences four forces:

- the normal force, in the upward direction,
- the force due to the acceleration of gravity, in the downward direction,
- the kinetic frictional force, opposite its direction of movement,
- and the tension force in the direction of motion.

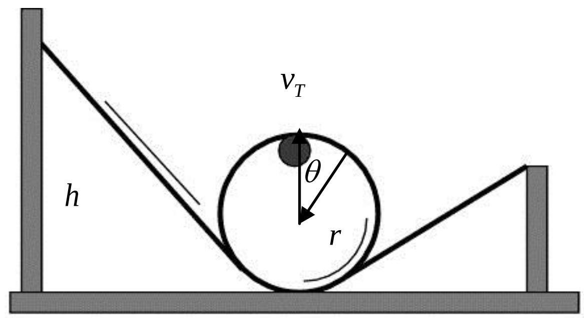


Part 3 Motion Along an Inclined Plane

The forces here are similar to those for an object moving vertically, however, the angle of the incline changes the magnitude of these forces



Part 4 Loop the Loop



The loop-the-loop is a simple device which we can use to study kinetic and potential energy. Motion of the ball down the track and around the loop-the-loop shows gravitational potential energy, rotational and translational kinetic energy, and centripetal force.

The ball must be released at some minimum height *h* above the bottom point of the track so that it will not leave the track while passing around the loop-the-loop. In order to stay on the track at the top of the loop the centripetal force of the ball on the track must be equal to or greater than the gravitational force on the ball.

III. Calculations

Part 1 Vertical Motion and Part 2 Horizontal Motion

Work is calculated using the equation:

$$W = Fd\cos\theta$$

where *F* is the applied constant force, *d* is the distance the object moves, and θ is the angle between the applied constant force and the direction in which the object is moving. When the force is applied in the same direction as the displacement:

$$W = Fd$$

The kinetic energy of an object is:

$$KE = \frac{1}{2}mv^2$$

where *m* is its mass, and *v* is its velocity.

The work-energy theorem combines these concepts to determine the work done by a net external force:

$$W = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

And, as a reminder, we can calculate the velocity of an object during a time *t* over a distance *d* using:

$$v = \frac{d}{t}$$

The net horizontal force on an object moving with constant velocity and zero acceleration in the *x* direction can be expressed as:

$$\sum F_x = 0$$

$$\sum F_x = T - f_k = 0$$

$$\sum F_x = T - \mu_k F_N = 0$$

$$\sum F_x = T - \mu_k mg = 0 \qquad \text{because } \sin\theta = 0$$

$$T = \mu_K F_N$$

The net vertical force on an object moving with constant velocity and zero acceleration in the *y* direction can be expressed as:

$$\sum F_y = 0$$

$$\sum F_y = F_N - W = 0$$

$$\sum F_y = F_N - mg = 0 \qquad \text{because } \cos\theta = 1$$

<u>Part 3 Motion Along an Inclined Plane</u>

We need to adjust these equations for constant velocity motion along an inclined plane with an angle θ:

$$\sum F_x = T - f_k - mg\sin\theta = 0$$

$$\sum F_x = T - \mu_k F_N - mg\sin\theta = 0$$

$$\sum F_y = F_N - mg\cos\theta = 0$$

We can then calculate the coefficient of kinetic friction:

$$T = \mu_k F_N + mg\sin\theta$$

$$F_N = mg\cos\theta$$

Then by substituting the equation for F_N in the equation for T:

$$T = \mu_k mg\cos\theta + mg\sin\theta$$

$$\mu_k = \frac{T - mg\sin\theta}{mg\cos\theta}$$

Remember that μ_k can't be negative, so if your calculations result in a negative number, check your measurements and redo the calculations.

Part 4 Loop the Loop

Conservation of Energy tells us that:

$$mgh = \frac{1}{2}mv^2 + mg(r + r\cos\theta)$$

where $mgh =$ initial potential energy before the ball is released, $\frac{1}{2}mv^2$ = kinetic energy of the ball in motion, $mg(r + r\cos\theta)$ = potential energy of the ball when it is some angle θ as measured from the vertical axis of the loop, and r = radius of the loop.

This simplifies to:

$$gh = \frac{1}{2}v^2 + g(r + r\cos\theta) \qquad \text{cancel } m \text{ from both sides}$$

and we can solve for the velocity of the ball at angle θ:

$$2gh = v^2 + 2g(r + r\cos\theta)$$

$$v^2 = 2gh - 2g(r + r\cos\theta)$$

$$v^2 = 2g[h - (r + r\cos\theta)]$$

$$v^2 = 2g[h - r(1 + \cos\theta)]$$

Top of Loop

When the ball is at the top of the loop, $\theta = 0^0$ and $\cos 0^0 = 1$, so the velocity v_t of the ball at the top of the loop is calculated from:

$$v_t^2 = 2g[h - r(1 + 1)]$$

$$v_t^2 = 2g(h - 2r)$$

$$v_t = \sqrt{2g(h - 2r)}$$

At the top of the loop, two forces act on it, the normal force, and the force as the result of the acceleration of gravity, which must be equal to the centripetal force, otherwise the ball falls off the track:

$$F_N + mg = \frac{mv_t^2}{r}$$

As the ball travels more slowly the normal force decreases until $F_N = 0$, meaning that gravity alone is sufficient to balance the required centripetal acceleration. At that time, the ball's critical velocity v_c is calculated from:

$$mg = \frac{mv_c^2}{r}$$

$$g = \frac{v_c^2}{r}$$

$$v_c^2 = gr$$

$$v_c = \sqrt{gr}$$

so when the ball has critical velocity at the top of the loop:

$$v_t = \sqrt{gr}$$

We want to find the critical height h_c such that $v_c = v_t$:

$$\sqrt{2g(h_c - 2r)} = \sqrt{gr}$$

$$2g(h_c - 2r) = gr$$

$$2(h_c - 2r) = r$$

$$2h_c - 4r = r$$

$$2h_c = 5r$$

$$h_c = \frac{5}{2}r$$

At the top of the loop the kinetic energy KE_t of the ball is:

$$KE_t = \frac{1}{2}mv_t^2$$

$$KE_t = \frac{1}{2}m(\sqrt{gr})^2$$

$$KE_t = \frac{1}{2}mgr$$

At the top of the loop the potential energy PE_t of the ball is:

$$PE_t = mg(2r)$$

$$PE_t = 2mgr$$

This also tells us, again, using the Law of Conservation of Energy, that for h_c:

$$mgh_c = KE_t + PE_t$$

$$mgh_c = \frac{1}{2}mgr + 2mgr$$

$$h_c = \frac{1}{2}r + 2r$$ cancel *mg* from both sides

$$h_c = \frac{5}{2}r$$

<u>Bottom of loop</u>

At the bottom of the loop all of the energy is kinetic, and from the Law of Conservation of Energy we can compute the ball's velocity at that point, keeping in mind that the initial kinetic energy of the ball is 0 and the potential energy of the ball at the bottom of the loop is also 0:

$$mgh = \frac{1}{2}mv_b^2$$ cancel *m* from both sides

$$gh = \frac{1}{2}v_b^2$$

$$v_b^2 = 2gh$$

$$v_b = \sqrt{2gh}$$

We can then compute the kinetic energy KE_b of the ball at the bottom of the loop:

$$KE_b = \frac{1}{2}mv_b^2$$

$$KE_b = \frac{1}{2}m(\sqrt{2gh})^2$$

$$KE_b = \frac{1}{2}m(2gh)$$

$$KE_b = mgh$$

IV. Equipment and Materials

Spring balances, washers, hooked mass containers, inclined plane, meter sticks, loop-the-loop apparatus, ball bearing, electronic balance, calculator, stop watch

V. Procedure

<u>Part 1 Vertical Motion</u>

1. Measure and record the distance from the floor to the countertop.

2. Record the mass of one of the supplied objects. Place the mass on the floor and attach it to a spring balance.

3. While one lab partner lifts the mass from the floor with uniform velocity, the other lab partner should time the lift and stop the watch when the bottom of the mass is level with the countertop. Record the time and the force on the spring balance.

4. Complete Table 1.

Part 2 Horizontal Motion

5. Measure a distance of at least one meter along the countertop.

6. While one lab partner pulls the mass along the measured distance at uniform velocity, the other partner times the pull. Record the time and tension on the spring balance.

7. Complete Table 2.

Part 3 Motion Along an Inclined Plane

8. Measure the vertical height *h* from the countertop to the top of the inclined plane. This is not the same *h* that you used in parts 1 and 2.

9. Use the same process as in Parts 1 and 2, but pull the mass up the inclined plane.

10. Complete Table 3.

Part 4 Loop the Loop

11. Experiment with the loop-the-loop to determine from what height the ball must be released to barely travel all the way around the loop.

12. Complete Table 4.

VI. Data

Table 1 Vertical Motion

A	B	C	D	E	F	G	H
Mass *m* of object in kg	Weight $F = mg$ of object in N	Height of countertop *h* in m	Time *t* to lift object in s	$W = Fh$ in joules	$v = \frac{h}{t}$ of object in m/s	$KE = \frac{1}{2}mv^2$ in joules	$PE = mgh$ at the end of the lift in joules

Table 2 Horizontal Motion

A	B	C	D	E	F	G	H
Mass *m* of object in kg	Tension $T = F$ of spring in N	Distance *d* in m	Time *t* to pull object in s	$W = Fd$ in joules	$v = \frac{d}{t}$ of object in m/s	$KE = \frac{1}{2}mv^2$ in joules	$PE = mgh$ at the end of the pull in joules

Table 3 Motion Along an Inclined Plane

A	B	C	D	E	F	G	H	I	J	K
Mass *m* of object in kg	Tension $T = F$ of spring in N	Height *h* of top of inclined plane in m	Distance *d* along inclined plane in m	Time *t* to pull object in s	Angle θ of inclined plane in degrees	Coef. of kinetic friction μ_k	$W = Fd$ in joules	$v = \frac{d}{t}$ of object in m/s	$KE = \frac{1}{2}mv^2$ in joules	$PE = mgh$ at the top of the incline in joules

Table 4 Loop-the-Loop

A	B	C	D	E	F	G	H
Mass *m* of ball in kg	Experimental critical height h_c in m	Loop radius *r* in m	Critical velocity $v_c = \sqrt{gr}$ in m/s	$KE_t = \frac{1}{2}mgr$ at top of loop in joules	$PE_t = 2mgr$ at the top of the loop in joules	$KE_b = mgh_c$ at bottom of loop in joules	PE_b at bottom of loop in joules

VII. Discussion Questions

Part 1 Vertical Motion

1. Draw a free body diagram of the situation, labeling all of the objects and forces.

Part 2 Horizontal Motion

2. Draw a free body diagram of the situation, labeling all of the objects and forces. How does this situation differ from that for vertical motion?

Part 3 Motion Along an Inclined Plane

3. Draw a free body diagram of the situation, labeling all of the objects and forces. How does this situation differ from those for horizontal and vertical motion?

Part 4 Loop the Loop

4. Calculate the critical height using $h_c = \frac{5}{2}r$ and compare it with the experimental height. What factors, other than the measurement of h_c might account for the difference?

5. Calculate the percent error between the experimental and critical heights:

$$\%\ error = \frac{experimental\ height - critical\ \ height}{critical\ \ height}\ x\,100\%$$

What is the percent error?

6. Why is the potential energy 0 joules at the bottom of the loop?

7. Why does the ball have both kinetic and potential energy at the top of the loop?