I. Objectives

1. Add and subtract vectors.
2. Determine the resultant vector mathematically and graphically.

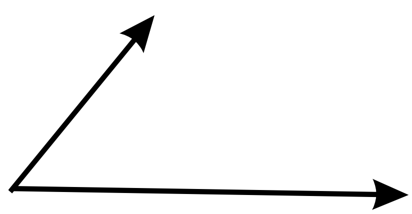
II. Introduction

One of the most important mathematical techniques in physics is vector algebra. The study of motion, electricity, magnetism, gravity, and light, and other concepts, require the use of vectors.

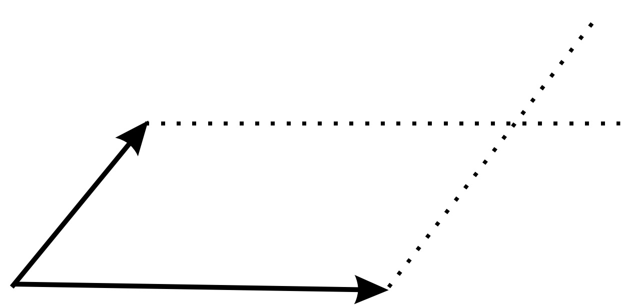
Anything which has both size and direction is a vector. Examples of vector quantities are force, acceleration, velocity, and momentum. Such things as mass, density, and volume have no direction and are called scalar quantities.

Addition of vectors can be accomplished in two ways: the geometric method, usually called the “parallelogram” or “head-to-tail” method, and the trigonometric method, which provides more accurate answers.

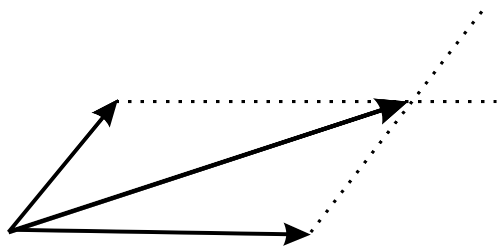
Using the geometric method two vectors to be added are drawn tail-to-tail:



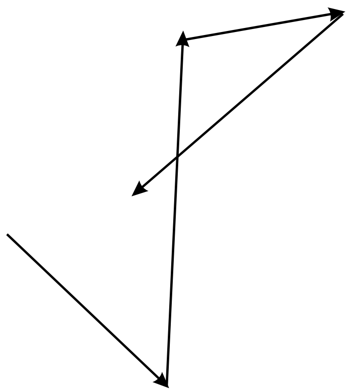
Next, lines are drawn from the head of each vector, parallel to the other:



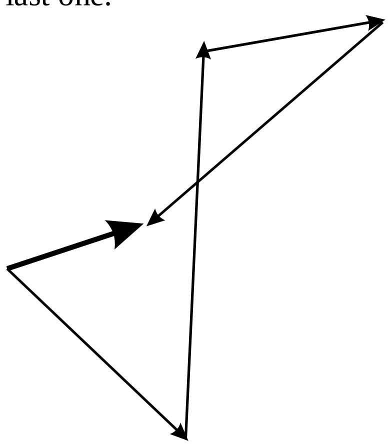
Finally, the vector sum is determined by drawing a vector from the intersection of the tails to the intersection of the parallel lines.



The head-to-tail method is similar but can be used to add more than two vectors at once. The vectors are drawn with the tail of each vector located at the head of the previous vector:



The vector sum is found by drawing a vector from the tail of the first vector to the head of the last one.



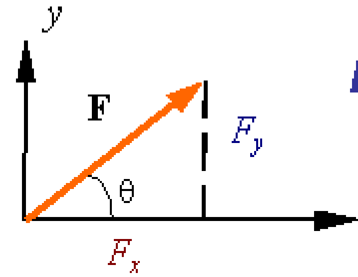
The magnitude and direction of the vector sum is found by measuring its length and angle using appropriate instruments. The accuracy of this method is limited by the care and accuracy of the drawing.

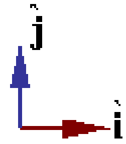
The trigonometric method involves the decomposition of each vector into x and y components. The sum of the individual x components provides the resultant x component, and the sum of the individual y components provides the resultant y component. The two components are then combined using vector algebra to determine the magnitude and direction of the resultant.

III. Equipment and Materials

Calculator

IV. Calculations





We begin by reviewing some vector basics:

Table 1 Vector Components and Angles

vector	magnitude	*x* component	*y* component	angle
A	$A = \sqrt{A_x^2 + A_y^2}$	$A_x = A\cos\theta_A$	$A_y = A\sin\theta_A$	$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right)$
B	$B = \sqrt{B_x^2 + B_y^2}$	$B_x = B\cos\theta_B$	$B_y = B\sin\theta_B$	$\theta_B = \tan^{-1}\left(\frac{B_y}{B_x}\right)$
R	$R = \sqrt{R_x^2 + R_y^2}$	$R_x = A_x + B_x$ $= R\cos\theta_R$	$R_y = A_y + B_y$ $= R\sin\theta_R$	$\theta_R = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

Note:

- **A** is ***not*** equal to $A_x + A_y$. To find **A**, its individual components A_x and A_y must be found then **A** can be calculated using the Pythagorean Theorem.
- **B** is ***not*** equal to $B_x + B_y$. To find **B**, its individual components B_x and B_y must be found then **B** can be calculated using the Pythagorean Theorem.
- **R** is ***not*** equal to **A** + **B**. To find **R**, its individual components R_x and R_y must be found then **R** can be calculated using the Pythagorean Theorem.
- θ_R is ***not*** equal to $\theta_A + \theta_B$. To find θ_R its individual components R_x and R_y must be found then θ_R can be calculated using $\tan^{-1}$.

V. Procedure

1. Complete Table 2. Vectors **A** and **B** are the components of resultant vector **R**.

VI. Data

Table 2 Vector Components and Resultant Vectors

	Vector components				
Trial	Vector	Magnitude in m	Angle θ in degrees	x component in m	y component in m
1	**A**	20.0	θ_A = 30.0	A_x =	A_y =
	B	35.0	θ_B = 60.0	B_x =	B_y =
	R		θ_R =	R_x =	R_y =
2	**A**		θ_A =	A_x = 12.0	A_y = 15.0
	B	18.0	θ_B = 42.0	B_x =	B_y =
	R		θ_R =	R_x =	R_y =
3	**A**	47.0	θ_A =	A_x = 29	A_y =
	B		θ_B = 57.0	B_x =	B_y =
	R		θ_R =	R_x = 71	R_y =
4	**A**		θ_A = 67.0	A_x =	A_y = 43.0
	B	59.0	θ_B =	B_x =	B_y =
	R		θ_R =	R_x =	R_y = 79.2
5	**A**		θ_A =	A_x =	A_y =
	B		θ_B = 62.5	B_x =	B_y = 24.6
	R	78.0	θ_R = 37.0	R_x =	R_y =

VII. Discussion Questions

1. Given the magnitude of vector **A**, and one of its components, A_x, what equation is required to calculate the value of the other component, A_y?

2. Given the magnitude of vector **B**, and angle θ_B, what equations are required to determine components B_x and B_y?

3. Given A_x and R_x what equation is required to determine B_x?

4. Given B_y and R_y what equation is required to determine A_y?

5. Given A_y and angle θ_A for vector **A**, what equation is required to determine A_x?