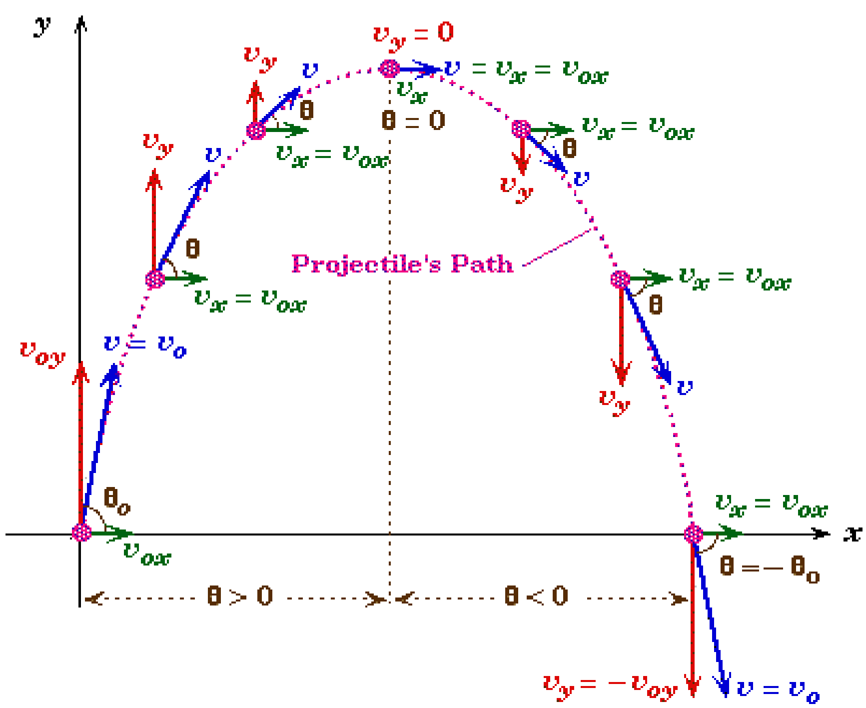
I. Objectives

1. Investigate the relationships among projectile initial velocity, launch angle, flight time, and vertical and horizontal displacements.
2. Determine how doubling the initial projectile velocity affects projectile flight parameters.

II. Introduction

A projectile is any object which once projected continues in motion by its own inertia and is influenced only by the downward force of gravity, assuming that air resistance is negligible. There are a variety of examples of projectiles, such as an object dropped from rest, an object thrown upwards, and an object thrown upwards at an angle. In this lab you will use a device that fires two metal balls.

Before beginning this lab you **must** work through the equations in the **Calculations** section below.



(Source: http://www.ac.wwu.edu/~vawter/PhysicsNet/Topics/Vectors/Gifs/ProjectileMotion01.gif)

III. Calculations

Projectile Variables

$x = x(t)$ = horizontal displacement at time t

x_{max} = maximum horizontal displacement, occurs when the launch angle $\theta = 45^0$

y_{max} = maximum height, where $v_y = 0$ m/s

v_0 = initial velocity

v_{0x} = initial velocity in the x direction = $v_0 \cos\theta$, we will assume that this is positive

v_{0y} = initial velocity in the y direction = $v_0 \sin\theta$

v_{ymax} = y velocity of projectile at maximum height = 0 m/s

t_{ymax} = time for projectile to reach maximum height y_{max}

t_{total} = projectile total flight time = $2t_{ymax}$

Horizontal Motion Projectile Equations

$x = x(t) = x_0 + v_{0x}\,t + \frac{1}{2}a_x t^2$ kinematics equation, horizontal displacement $x(t)$ at time t

$x_0 = 0\ m$ assume initial horizontal displacement = 0 m

$a_x = 0\ m/s^2$ assume horizontal velocity is constant

$x = x(t) = v_{0x}\,t$ simplification

$v_{0x} = v_0 \cos\theta$ from above

$x = x(t) = (v_0 \cos\theta)\,t$ substitution

$t = t(x) = \frac{x}{v_0 \cos\theta}$ solving for $t = t(x)$, time t at horizontal displacement x

Vertical Motion Projectile Equations

$y(t) = y_0 + v_{0y}\, t + \frac{1}{2} a_y t^2$ kinematics equation, vertical displacement *y(t)* at time *t*

$y_0 = 0\ m$ assume initial vertical displacement = 0 m

$a_y = -g$ acceleration in the *y* direction due only to gravity,
vertical acceleration is constant,
vertical velocity is **NOT** constant

$v_{0y} = v_0 \sin\ \theta$ from above

$y(t) = v_{0y}\ t + \frac{1}{2} a_y t^2$ simplification

$y(t) = v_0 (\sin\ \theta) t - \frac{1}{2} g t^2$ substitution, vertical displacement at time *t*

$t = t(x) = \frac{x}{v_0 \cos \theta}$ from above, time *t* at horizontal displacement *x*

$$y(x) = v_0 (\sin\ \theta) \left(\frac{x}{v_0 \cos \theta} \right) - \frac{1}{2} g \left(\frac{x}{v_0 \cos \theta} \right)^2$$
substitution for *t*

$$y(x) = (\sin\ \theta) \left(\frac{x}{\cos\ \theta} \right) - \frac{1}{2} g \left(\frac{x}{v_0 \cos\ \theta} \right)^2$$
simplification

$$y(x) = (x) \left(\frac{\sin \theta}{\cos \theta} \right) - \frac{1}{2} \frac{g x^2}{{v_0}^2} \left(\frac{1}{\cos^2\ \theta} \right)$$
rewrite

$\tan\ \theta = \frac{\sin\ \theta}{\cos \theta}$ definition of tangent

$\sec\ \theta = \frac{1}{\cos \theta}$ definition of secant

$$\sec^2\theta = \frac{1}{\cos^2\theta}$$ squaring both sides of the equation above

$$y(x) = x\tan\theta - \frac{gx^2\sec^2\theta}{2v_0^2}$$

substitution, vertical displacement y at horizontal displacement x

Solving for the Horizontal Displacement Using the Quadratic Formula

The horizontal displacement traveled by a projectile at its landing location is often referred to as its range, denoted by R. We can determine this displacement using the quadratic formula.

$$y(x) = \left(\frac{-g\sec^2\theta}{2v_0^2}\right)x^2 + (\tan\theta)x$$

from above, rearrange the equation above as a quadratic equation

$$y(x) = Ax^2 + Bx + C$$ general equation for a parabola

$$A = \left(\frac{-g\sec^2\theta}{2v_0^2}\right)$$ substitution

$$B = (\tan\theta)$$ substitution

$$C = 0\ m$$ substitution

$$y(x) = 0\ m$$ when the projectile is on the ground it is either at its starting or landing location, meaning that its height y = 0 m

$$0\ m = Ax^2 + Bx + C$$ substitution

$$0\ m = \left(\frac{-g\sec^2\theta}{2v_0^2}\right)x^2 + (\tan\theta)x$$

determine the range R, when the projectile is on the ground, at its landing location

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$ quadratic formula

$$x = \frac{-B \pm \sqrt{B^2}}{2A}$$ simplification, $C = 0$ m

$$x = \frac{-\tan\theta \pm \sqrt{\tan^2\theta}}{2\left(\frac{-g\sec^2\theta}{2v_0^2}\right)}$$ substitution

$$x = \frac{-\tan\theta \pm \tan\theta}{\left(\frac{-g\sec^2\theta}{v_0^2}\right)}$$ simplification

$$x = \frac{(-\tan\theta \pm \tan\theta)v_0^2}{-g\sec^2\theta}$$ simplification

$$x = 0\ m$$ for the + case

$$x = \frac{(-2\tan\theta)v_0^2}{-g\sec^2\theta}$$ for the - case

$$x = \frac{(2\tan\theta)v_0^2}{g\sec^2\theta}$$ simplification of the - case

$$x = \frac{\left(2\frac{\sin\theta}{\cos\theta}\right)v_0^2}{g\left(\frac{1}{\cos^2\theta}\right)}$$ rewrite, using the trigonometric definitions from above

$$x = \frac{2v_0^2(\sin\theta\cos\theta)}{g}$$ simplification

$$x = R = \frac{2v_0^2\sin 2\theta}{g}$$ substitution, $\sin 2\theta = 2\cos\theta\sin\theta$

Case #1: Maximum Height y_{max}

$v_{ymax} = 0$ m/s	at maximum height y_{max} the projectile has stopped moving in the vertical direction
$v_y^2 = v_{0y}^2 - 2gy$	kinematics equation
$v_{0y} = v_0 \sin\theta$	from above
$v_{y\max}^2 = v_{0y}^2 - 2gy_{\max}$	substitution
$0^2 m^2/s^2 = (v_0 \sin\theta)^2 - 2gy_{\max}$	substitution
$2gy_{\max} = (v_0 \sin\theta)^2$	rearrange the equation above
$2gy_{\max} = v_0^2 \sin^2\theta$	simplification
$y_{\max} = \dfrac{v_0^2 \sin^2\theta}{2g}$	solve for y_{max}
$y_{\max} = \dfrac{(v_0 \sin\theta)^2}{2g}$	will be used in Case #2, below

Case #1: Time t_{ymax} at Maximum Height y_{max}

$v_{ymax} = 0$ m/s	y velocity of projectile at maximum height = 0 m/s
$v_{y\max} = v_{oy} - gt_{y\max}$	kinematics equation and substitution
$v_{y\max} = v_0 \sin\theta - gt_{y\max}$	substitution
$0\, m/s = v_0 \sin\theta - gt_{y\max}$	substitution
$gt_{y\max} = v_0 \sin\theta$	rearrange the equation above
$t_{y\max} = \dfrac{v_0 \sin\theta}{g}$	solve for t_{ymax}

Case #1: Total Travel Time t_{total}

$t_{total} = 2t_{y\max}$	multiply the time to reach maximum height by 2
$t_{total} = \dfrac{2v_0 \sin\theta}{g}$	substitution

Case #1: Initial Velocity v_0

$v_0 = \dfrac{t_{total}g}{2\sin\theta}$	from the equation above

Case #1: Horizontal Displacement x

$x = (v_{0x})(t_{total})$	displacement = (rate)(time)
$v_{0x} = v_0 \cos\theta$	from above
$x = (v_0 \cos\theta)\, t_{total}$	subsitution
$x = (v_0 \cos\theta)\left(\dfrac{2v_0 \sin\theta}{g}\right)$	substitution of t_{total}
$x = \dfrac{v_0^2 (2\cos\theta \sin\theta)}{g}$	simplification
$\sin 2\theta = 2\cos\theta\sin\theta$	from trigonometry
$x = \dfrac{v_0^2 \sin 2\theta}{g}$	substitution

Launch Angle θ that Produces the Maximum Range, x_{max} = R

$\dfrac{dx}{d\theta} = \dfrac{2v_0^2 \cos\ 2\theta}{g} = 0$	from calculus
$\cos\ 2\theta = 0\left(\dfrac{g}{2v_0^2}\right)$	rearrange the equation above
$\cos\ 2\theta = 0$	simplification

$\cos 90 = 0$ from trigonometry

$\cos 2\theta = \cos 90^0$ substitution

$2\theta = 90^0$ simplification

$\theta = 45^0$ maximum possible horizontal displacement x_{max} = R

Case #2: Maximum Height y_{max}

$$y_{\max} = \frac{(2v_0)^2 \sin^2 \theta}{2g}$$

assume the initial velocity is $2v_0$, twice what it was in Case #1, replace $v_0 \sin\theta$ in the results for Case #1 with $2v_0 \sin\theta$

$$y_{\max} = \frac{4v_0^2 \sin^2 \theta}{2g}$$

simplification

$$y_{\max} = \frac{2v_0^2 \sin^2 \theta}{g}$$

simplification

Case #2: Time t_{ymax} at Maximum Height y_{max}

$$t_{y\max} = \frac{2v_0 \sin\theta}{g}$$

replace v_0 with $2v_0$, using the equation for t_{ymax} from Case #1 above

Case #2: Total Travel Time t_{total}

$$t_{total} = \frac{4v_0 \sin \theta}{g}$$

multiply the time to reach maximum height by 2

Case #2: Initial Velocity v_0

$$v_0 = \frac{t_{total} g}{4 \sin \theta}$$

from the equation above

<u>Case #2: Horizontal Displacement *x*</u>

$x = (v_{0x})(t_{total})$	displacement = (rate)(time)
$v_{0x} = 2v_0 \cos\theta$	substitution
$x = (2v_0 \cos\theta)\, t_{total}$	substitution
$x = (2v_0 \cos\theta)\left(\dfrac{4v_0 \sin\theta}{g}\right)$	substitution of t_{total}
$x = \dfrac{8v_0^2 \cos\theta \sin\theta}{g}$	simplification
$x = \dfrac{4v_0^2 (2\cos\theta \sin\theta)}{g}$	rewrite the equation above
$x = R = \dfrac{4v_0^2 \sin 2\theta}{g}$	using the trigonometric identity $\sin 2\theta = 2\cos\theta \sin\theta$ yields the range $x = R$

We can now compare the results from Case #1 with Case #2:

Table 1 Projectile for Case #1 and Case #2

A	B	C	D	E	F
Case	Initial velocity	Maximum height y_{max}	Time t_{ymax} to reach y_{max}	Total time t_{total}	Horizontal displacement x = range R
#1	v_0	$\dfrac{v_0^2 \sin^2\theta}{2g}$	$\dfrac{v_0 \sin\theta}{g}$	$\dfrac{2v_0 \sin\theta}{g}$	$\dfrac{v_0^2 \sin 2\theta}{g}$
#2	$2v_0$	$\dfrac{2v_0^2 \sin^2\theta}{g}$	$\dfrac{2v_0 \sin\theta}{g}$	$\dfrac{4v_0 \sin\theta}{g}$	$\dfrac{4v_0^2 \sin 2\theta}{g}$
Multiply Case #1 by to get Case #2	2	4	2	2	4

Note that in column C of Table 1 the initial velocity is squared to calculate the maximum height and in column F the initial velocity is squared to calculate the horizontal displacement.

IV. Equipment and Materials

Calculator, projectile motion device, metal balls, meter sticks, calculator

V. Procedure

<u>Part 1: Comparison of Case #1 and Case #2</u>

1. Answer the Discussion Questions for this part of the lab, below.

<u>Part 2: Projectiles</u>

2. Read the remaining Discussion Questions **BEFORE** you begin. You will need to answer some of them as you work through the lab.

3. You may use only a meter stick to measure horizontal range R and vertical height H of the metal balls.

4. Horizontal range R is the total horizontal displacement traveled by the ball with the **non-zero horizontal velocity**. Vertical height H is the total vertical displacement traveled by the balls. You will **calculate** all remaining data.

5. Be sure that you use the correct units in your answers. **No units or incorrect units, no credit**.

6. Assume the initial vertical velocities of both balls are 0.00 m/s and that the horizontal velocity of both balls is constant. This does **not** mean that the horizontal velocities of the balls are the same.

7. Use the value of -9.81 m/s^2 for g so that you don't need to remember to include negative signs in the equations.

8. The device provided in class will "fire" two balls, allowing one to fall directly to the floor. The other will have a **non-zero horizontal** velocity.

9. "Fire" the balls and observe their landing.

10. Measure the height H in **meters** traveled by the balls. **<u>Remember that the vertical height H is negative</u>.** Write this number in the last row of Table 2, column B and the last row of Table 3, column E.

    height H = _____ meters, vertical displacement traveled by the balls

11. Measure the horizontal range R in **meters** of the ball with non-zero horizontal velocity. **Remember that the horizontal range R is positive.** Write this number is the last row of Table 2, column E and the last row of Table 3, column B.

    Range R = _____ meters, horizontal displacement traveled by the ball with **non-zero** horizontal velocity

12. Complete Table 2. Remember that the balls are accelerating in the vertical direction so you cannot, for example, simply halve the total time to calculate the time for the balls to travel half the horizontal displacement. Hints: Calculate column B first. What do you know about the values of Table 2, column F?

13. Complete Table 3. Hints: Calculate Table 3, column B first. What do you know about the values of Table 3, column D?

14. Complete Table 4. Write the x values from Table 2, column E and Table 3, column B in Table 4, column B. Write the corresponding y values, paired with those x values, from Table 2, column B and Table 3, column E in Table 4, column C.

15. **Calculate** the numeric value of t, the time it takes for the balls to land. Write this number in the last row of Table 2, column C and the last row of Table 3, column C. You may need to complete several of the Discussion Questions **before** calculating this value.

    t = _____ seconds, total time

16. Complete Table 5. The first row, where x = 0.00 m and y = 0.00 m is the starting point in our problem. Sort the x values in Table 4 from lowest to highest and write them in Table 5, column A. Write their corresponding y values in Table 5, column B. There will be duplicate data. Include it.

17. Complete Table 6. In Table 6, column D write the vector $\vec{v}$ for the ball with the non-zero x velocity. The vector should be in the form of $\vec{v} = v_x\hat{x} + v_y\hat{y}$ where v_x and v_y are replaced with the values you calculated in previous steps and/or tables. Calculate the velocity magnitude $v = \sqrt{v_x^2 + v_y^2}$ for the ball with the non-zero x velocity and write the results in Table 6, column E. Calculate the fall angle θ for the ball with the non-zero x velocity and write the result in Table 6, column F.

18. Graph the data from Table 5, columns A and B **and the polynomial order 2 best fit line** generated by the spreadsheet program. You must include labeled and appropriately scaled axes, correct units, a correct graph title, and the correct best fit line to receive credit. Staple the graph to this lab.

VI. Data

Table 2 Horizontal and Vertical Ball Parameters Using Vertical Displacements

A	B	C	D	E	F
ball has fallen	**y** calculated vertical displacement in m	calculated time *t* in s to fall indicated displacement	calculated velocity v_y in m/s at displacement *y*	**x** calculated horizontal displacement in m at time *t*	calculated velocity v_x in m/s at displacement *x*
1/4 of height H					
1/2 of height H					
3/4 of height H					
height H					

Table 3 Horizontal and Vertical Ball Parameters Using Horizontal Displacements

A	B	C	D	E	F
ball has traveled	**x** calculated horizontal displacement in m	calculated time *t* in s to travel indicated displacement	calculated velocity v_x in m/s at displacement *x*	**y** calculated vertical displacement in m at time *t*	calculated velocity v_y in m/s at displacement *y*
1/4 of range R					
1/2 of range R					
3/4 of range R					
range R					

Table 4 Ball x and y Position Coordinates

A	B	C
ball position	**x** in m	**y** in m
1/4 of height H		
1/2 of height H		
3/4 of height H		
height H		
1/4 of range R		
1/2 of range R		
3/4 of range R		
range R		

Table 5 Sorted Ball x and y Position Coordinates and Calculated y Values

A	B	C	D
sorted **x** values in m	corresponding **y** values in m	x^2 in m^2	calculated $y = \frac{-gx^2}{2v_x^2}$ in m
0.00	0.00		

Table 6 Vector Components

A	B = Table 3D	C = Table 2D	D	E	F
ball has fallen:	calculated velocity v_x at displacement x in m/s	calculated velocity v_y at displacement y in m/s	calculated velocity vector $\vec{v} = v_x\hat{x} + v_y\hat{y}$	calculated velocity magnitude $v = \sqrt{v_x^2 + v_y^2}$ in m/s	calculated fall angle $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$ in degrees
¼ of height H					
½ of height H					
¾ of height H					
height H					

VII. Discussion Questions

Part 1: Comparison of Case #1 and Case #2

1. At what launch angle θ is the maximum horizontal displacement, or range of the projectile, achieve its greatest numeric value?

2. What is the vertical velocity when a projectile is at its maximum height? Explain.

3. How does doubling the velocity from v_0 to $2v_0$ affect the maximum height y_{max}? By what specific numerical factor is it increased? Hint: see Table 1 above.

4. How does doubling the velocity from v_0 to $2v_0$ affect the time t_{ymax} to reach its maximum height y_{max}? By what specific numerical factor is it increased?

5. How does doubling the velocity from v_0 to $2v_0$ affect the range R, the maximum horizontal displacement? By what specific numerical factor is it increased?

6. How do you calculate the total travel time t_{total} of the projectile if you know the time t_{ymax} to reach its maximum height y_{max}?

7. How would tripling the velocity from v_0 to $3v_0$ affect the maximum height y_{max}? By what specific numerical factor would it be increased? Hint: see the note below Table 1.

8. How would tripling the velocity from v_0 to $3v_0$ affect the time t_{ymax} to reach its maximum height y_{max}? By what specific numerical factor would it be increased?

9. How would tripling the velocity from v_0 to $3v_0$ affect the range x, the horizontal distance? By what specific numerical factor would it be increased?

Part 2: Projectiles

10. Which one of the two balls do you and your team members think will land first? Explain your reasoning.

11. What actually happened when you fired the balls? Did they land as you expected? Explain why or why not.

12. Explain why the numeric value of the **vertical acceleration** for each of the two balls is the same. What is the numeric value of a_y? Why is it negative?

13. What is the numeric value of the **initial vertical velocity** v_{0y} of the balls? Hint: read the instructions.

    v_{0y} = _____ meters/second

14. Write the equation to calculate the **vertical displacement** y at time t. Hint: start with the kinematics equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to calculate y and **simplify as much as possible**.

15. Solve the equation in the step directly above for time t, to reach fall displacement y. Neatly show your work.

16. Describe the shape of the path of the ball with the **non-zero horizontal velocity**. "Curved" is not the correct answer. You must be more specific.

17. Are the balls accelerating in the **horizontal** direction? Explain. Hint: read the instructions.

18. What is the numeric value of the horizontal velocity for the ball falling straight down?

    v_{xdown} = _____ meters/second

19. Is the numeric value of the horizontal velocity v_x for the ball with the **non-zero horizontal** velocity negative or positive? Explain.

20. Write the equation to calculate the horizontal displacement x at time t for the ball with a non-zero horizontal velocity v_x. Hint: start with the kinematics equation $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ and simplify as much as possible.

21. Solve the equation in the step directly above for time t.

22. Solve the equation in the step directly above for the initial horizontal velocity v_{0x}.

23. In two previous steps, you derived two different equations for t, one involving x and the other involving y. Set them equal to each other and solve for y then replace v_{0x} with v_x. Neatly show your work.

24. Demonstrate that $\frac{-g}{2v_x^2}$ is in units of $\frac{1}{meters}$. Include the units for each variable and neatly show your work.

25. Demonstrate that $\frac{-gx^2}{2v_x^2}$ is in units of meters. Include the units for each variable and neatly show your work.

26. Calculate the numeric value of $\frac{-g}{2{v_x}^2}$. Neatly show your work.

27. Compare the numbers in Table 5, column B and Table 5, column D. They should be the same. Explain why. If they are not the same, you will need to redo your calculations.

28. Explain why θ is negative. Your answer must include a physical justification.