I. Objectives

1. Observe rate at which an object cools from a temperature that is higher than that of its environment.
2. Investigate the relationship between time and temperature and Newton's Law of Cooling.
3. Calculate the equation of a line using least squares analysis.

II. Introduction

This lab utilizes glassware and hot water. You must wear closed-toed shoes during this entire lab and goggles when working with hot water.

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the temperature of its environment.

This law describes an instantaneous rate of change of the temperature. We can translate this verbal statement into a differential equation, which we can transform into an algebraic equation that we can use to investigate a relationship between time and temperature.

III. Calculations

Newton's Law of Cooling states that the rate of change of the temperature $\frac{dT(t)}{dt}$ is proportional to the difference between the temperature of the substance $T(t)$ and the temperature of the environment T_e. Mathematically this can be written as a differential equation:

$$\frac{dT(t)}{dt} = -k[T(t) - T_e]$$

This differential equation can be solved to express a relationship we can use in this experiment:

$$T(t) = T_e + (T_0 - T_e)e^{-kt}$$

We can rewrite this equation in other forms:

$$T(t) - T_e = (T_0 - T_e)e^{-kt}$$

$$e^{-kt} = \frac{T(t) - T_e}{T_0 - T_e}$$

$$\ln\left(e^{-kt}\right) = \ln\left(\frac{T(t) - T_e}{T_0 - T_e}\right)$$

$$-kt = \ln\left(\frac{T(t) - T_e}{T_0 - T_e}\right)$$

$$-\ln\left(\frac{T(t) - T_e}{T_0 - T_e}\right) = kt$$

where

$T(t)$ = temperature at time t in C^0

T_0 = initial temperature of the substance in C^0

T_e = temperature of the environment in C^0

k = cooling constant

This equation is the equation of a line in the form of $y = mx + b$, where

$$y = -\ln\left(\frac{T(t) - T_e}{T_0 - T_e}\right)$$

$x = t$

slope $m = k$

intercept $b = 0$ because we are starting the experiment at $t = 0$.

To determine k, we will use a process called least square analysis. This method was used to determine the equation of a straight line before computers were used. The derivations for the equations are somewhat complicated, so we'll just use the results.

The slope m is:

$$m = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$$

and the y–intercept b is:

$$b = \bar{y} - m\bar{x}$$

The equation of the line is:

$$\bar{y} = m\bar{x} + b$$

or simply

$$y = mx + b$$

In this experiment we will also assume that the beaker is perfectly cylindrical. The volume of water V in the beaker is:

$$V = \pi r^2 h$$

where

r = radius of the beaker

h = height of the water in the beaker

The surface area A is:

$$A = 2\pi rh + 2\pi r^2$$

As the height h increases the volume V changes according to the equation:

$$\Delta V = \pi r^2 \Delta h$$

and the surface area A changes according to the equation:

$$\Delta A = 2\pi r \Delta h$$

IV. Equipment and Materials

600 mL beaker, hot plate, goggles, rubber grippers, water, thermometer, stop watch, calculator

V. Procedure

1. Record the temperature T_e of the room in Table 1 below.

2. Put approximately 250 mL of water and a thermometer in the 600 mL beaker. On a hot plate, heat the water uniformly to about 80^0C. Don't allow too much of the water to evaporate. Add more if needed.

3. Record the initial temperature T_0 of the water, which will be the first number in column B of Table 1, and start the stop watch.

4. Immediately turn off the hot plate, remove the beaker *carefully*, with the rubber grippers, and place it on the lab bench.

5. Record the time and temperature in Table 1 approximately every 3 minutes until you have collected 10 data points.

6. Graph Experimental $T(t)$ versus *t*, using *t* as the x-coordinate and Experimental $T(t)$ as the y-coordinate. You must include the exponential (not linear) best fit line generated by the spreadsheet program, not a hand drawn line, to connect the points on the graph.

7. Complete Tables 1, 2, and 3. You will have to complete some of the Discussion Questions as you complete the tables.

8. On another graph, plot $-\ln\left(\frac{T(t)-T_e}{T_0-T_e}\right)$ versus *t*, using *t* as the x-coordinate and $-\ln\left(\frac{T(t)-T_e}{T_0-T_e}\right)$ as the y-coordinate. This should result in a straight line with a slope approximately equal to *k*. You must include the linear (not exponential) best fit line generated by the spreadsheet program, not a hand drawn line, to connect the points on the graph. Staple the graph to the lab.

9. Complete Table 4**. You must use the value of *k* that you calculated in Table 3, column C to compute Table 4, column C**.

10. On the same graph on which you graphed Experimental *T(t)* versus *t*, graph Calculated *T(t)* versus *t*, using *t* as the x-coordinate and Calculated $T(t)$ as the y-coordinate. You must include the exponential (not linear) best fit line generated by the spreadsheet program, not a hand drawn line, to connect the points on the graph. Staple the graph to the lab.

11. Measure the radius *r* of the beaker and record the result in Table 5.

12. **Calculate** the height *h* the water would have in the beaker given the volumes in Table 5.

13. Complete Table 5. Hint: what is the relationship between mL and cm^3?

VII. Data

Table 1 Time and Temperature

A	B	C	D	E	F = D/E	G = -ln(F)
Time t in s	Experimental water temperature $T(t)$ in ^0C	Room temperature T_e in ^0C	$T(t)-T_e$ in C^0	T_0-T_e in C^0	$\frac{T(t)-T_e}{T_0-T_e}$ in C^0	$-\ln\left(\frac{T(t)-T_e}{T_0-T_e}\right)$
0.0	$T_0 =$					

Table 2 Preliminary Least Squares Analysis

A	B = A - $\bar{x}$	C = B^2	D	E = D - $\bar{y}$	F
x_i from Table 1 column A	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$	y_i from Table 1 column G	$(y_i-\bar{y})$	$(x_i-\bar{x})(y_i-\bar{y})$
$\bar{x} =$			$\bar{y} =$		

Table 3 Least Squares Data and Calculations

A	B	C	D	E
$\sum(x_i - \bar{x})(y_i - \bar{y})$	$\sum(x_i - \bar{x})^2$	$m = k = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$	y-intercept $b = \bar{y} - m\bar{x}$	Equation of line $y = mx + b$

Table 4 Experimental and Calculated Temperatures

A	B	C	D = B – C
Time *t* in *s* from Table 1 column A	Experimental water temperature $T(t)$ in ^0C from Table 1 column B	Calculated water temperature $T(t)$ in ^0C using $T(t) = T_e + (T_0 - T_e)e^{-kt}$	Difference between experimental and calculated values in C^0
0.0			

Table 5 Volume and Surface Area

A	B	C	D	E	F = B/E
Water volume *V* in mL	Water volume *V* in cm^3	Measured beaker radius *r* in cm	Calculated water height *h* in cm	Calculated surface area *A* in cm^2	Water volume V / Surface area A in cm
100					
200					
300					
400					
500					

VIII. Discussion Questions

1. Assuming that you used exactly 250 mL of water, how much heat would be lost by the water as it cooled from T_0 to T_e? Include your calculations. Hint: how are the mass and volume of water related?

2. What is the slope *m* of the line? Remember that this is your experimental value for *k* that is used to calculate the values in column C of Table 3 and in the following question.

3. Substitute the values of T_0, T_e and *k* into the equation $T(t) = T_e + (T_0 - T_e)e^{-kt}$ and simplify as much as possible. Do not substitute values for *T(t)* or *t*.

4. Using this equation, how long will it take for the water in this experiment to almost reach room temperature? Hint: because this is an exponential equation, we can't solve it for $T(t) = T_e$, so assume that we are looking for the **time *t*** it takes to reach 1^0C above room temperature, $T(t) = T_e + 1^0$C. Include the calculations you used to answer the question.

5. How does the volume of water affect the volume to surface ratio?

6. Would *k* be different than the *k* you calculated in this experiment using the 600 mL beaker with different volumes water? Explain why or why not, and if so, how it would be different. Hint: review the results in Table 5.

7. Would *k* be different than the *k* you calculated in this experiment if a different substance were used instead of water? Explain why or why not, and if so, how it would be different.

8. What other factors could affect the value of k? Why?