VII. Discussion Questions

1. Which object, the disk or the ring, has greater translational speed at the bottom of the incline? Provide a mathematical explanation to support your answer.

   The disk has greater translational speed and arrives at the bottom first because

$$\sqrt{\frac{4}{3}gh_0} > \sqrt{gh_0} \quad \rightarrow \quad 1.155\sqrt{gh_0} > \sqrt{gh_0} \quad \rightarrow \quad v_{fdisk} > v_{fring}$$

2. Simplify the ratio $\frac{v_{fdisk}}{v_{fring}}$ as much as possible and find its numerical value.

$$\frac{v_{fdisk}}{v_{fring}} = \frac{\sqrt{\frac{4}{3}gh_0}}{\sqrt{gh_0}} = \sqrt{\frac{4}{3}} = 1.155$$

3. What factors could explain the differences between the experimental and calculated velocities at the bottom of the incline?

   Releasing the disk and ring at slightly different times, human error in determining the time, friction.

4. Using the conditions listed in the beginning of the Calculations section, derive the velocity equation for a solid sphere with its axis through the center and simplify as much as possible.

$$\frac{1}{2}mv_{fsphere}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega_{fsphere}^2 = mgh_0 \quad \rightarrow \quad \frac{1}{2}v_{fsphere}^2 + \left(\frac{1}{5}r^2\right)\omega_{fsphere}^2 = gh_0 \quad \rightarrow$$

$$\frac{1}{2}v_{fsphere}^2 + \left(\frac{1}{5}r^2\right)\left(\frac{v_{fsphere}}{r}\right)^2 = gh_0 \quad \rightarrow \quad \frac{1}{2}v_{fsphere}^2 + \frac{1}{5}v_{fsphere}^2 = gh_0 \quad \rightarrow$$

$$\frac{7}{10}v_{fsphere}^2 = gh_0 \quad \rightarrow \quad v_{fsphere}^2 = \frac{10}{7}gh_0 \quad \rightarrow$$

$$v_{fsphere} = \sqrt{\frac{10}{7}gh_0} \quad \rightarrow \quad v_{fsphere} = 1.195\sqrt{gh_0}$$

5. Using the conditions listed in the beginning of the Calculations section, derive the velocity equation for a spherical shell with its axis through the center and simplify as much as possible.

$$\frac{1}{2}mv_{fshell}^2+\frac{1}{2}\left(\frac{2}{3}mr^2\right)\omega_{fshell}^2=mgh_0 \rightarrow \frac{1}{2}v_{fshell}^2+\left(\frac{1}{3}r^2\right)\omega_{fshell}^2=gh_0 \rightarrow$$

$$\frac{1}{2}v_{fshell}^2+\left(\frac{1}{3}r^2\right)\left(\frac{v_{fshell}}{r}\right)^2=gh_0 \rightarrow \frac{1}{2}v_{fss}^2+\frac{1}{3}v_{fss}^2=gh_0 \rightarrow$$

$$\frac{5}{6}v_{fshell}^2=gh_0 \rightarrow v_{fshell}^2=\frac{6}{5}gh_0 \rightarrow$$

$$v_{fshell}=\sqrt{\frac{6}{5}gh_0} \rightarrow v_{fshell}=1.095\sqrt{gh_0}$$

6. Simplify the ratio $\frac{v_{fsphere}}{v_{fshell}}$ as much as possible and find its numerical value.

$$\frac{v_{sphere}}{v_{fshell}}=\frac{\sqrt{\frac{10}{7}gh_0}}{\sqrt{\frac{6}{5}gh_0}}=\sqrt{\left(\frac{10}{7}\right)\left(\frac{5}{6}\right)}=\sqrt{\frac{50}{42}}=\sqrt{\frac{25}{21}}=1.091$$

7. Compare the ratios $\frac{v_{fdisk}}{v_{fring}}$ and $\frac{v_{fsphere}}{v_{fshell}}$. What similarities are there between the velocities and the mass distributions for these two cases?

In both cases the objects whose masses are evenly distributed have a greater final velocity than those whose mass is concentrated only on the surface.