I. Objectives

1. Investigate the relationships among moment of inertia, mechanical energy, final velocity, and mass distribution.

II. Introduction

Rotational and translational motion often occur within the same physical systems. In this lab, we'll investigate how the mass distribution in two rotating objects, a solid cylinder or disk, and a hollow cylinder or ring, with the same mass and same radius affects their rolling velocities down an incline.

III. Calculations

We will make the following assumptions for the disk and the ring:

$m = m_{disk} = m_{ring}$	the masses of the disk and the ring are equal
$r = r_{disk} = r_{ring}$	the radii of the disk and the ring are equal
$v_0 = v_{0disk} = v_{0ring} = 0$ m/s	initial velocity
$h_0 = h_{0disk} = h_{0ring}$	initial height
$h_f = h_{fdisk} = h_{fring} = 0$ m	final height
$\omega_0 = \omega_{0disk} = \omega_{0ring} = 0\ rad/s$	initial angular velocity

From the Law of Conservation for Mechanical Energy:

$$E_f = E_0$$

$$E_0 = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + mgh_0$$

$$E_f = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f$$

$$\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + mgh_0$$

$$\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 = mgh_0$$ eliminating terms equal to 0

Keeping in mind that $m = m_{disk} = m_{ring}$ and $r = r_{disk} = r_{ring}$, we determine the moment of inertia, I, based on the shape of the object:

$$I = \frac{1}{2}m_{disk}r_{disk}^2 = \frac{1}{2}mr^2$$ disk moment of inertia

$$I = m_{ring}r_{ring}^2 = mr^2$$ ring moment of inertia

To determine the calculated final velocity v_{fdisk} for the disk:

$$\frac{1}{2}mv_{fdisk}^2 + \frac{1}{2}I\omega_{fdisk}^2 = mgh_0$$

$$\frac{1}{2}mv_{fdisk}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_{fdisk}^2 = mgh_0$$ substitution

$$\frac{1}{2}mv_{fdisk}^2 + \left(\frac{1}{4}mr^2\right)\omega_{fdisk}^2 = mgh_0$$

$$v_{fdisk}^2 + \frac{1}{2}r^2\omega_{fdisk}^2 = 2gh_0$$ multiplying both sides by 2 and cancelling out m

$$v_{fdisk} = \omega_{fdisk}r$$

$$\omega_{fdisk} = \frac{v_{fdisk}}{r}$$

$$v_{fdisk}^2 + \frac{1}{2}r^2\left(\frac{v_{fdisk}}{r}\right)^2 = 2gh_0$$ substitution

$$v_{fdisk}^2 + \frac{1}{2}v_{fdisk}^2 = 2gh_0$$ simplify

$$\frac{3}{2}v_{fdisk}^2 = 2gh_0$$

$$v_{fdisk}^2 = \frac{4}{3} g h_0$$

$$v_{fdisk} = \sqrt{\frac{4}{3} g h_0}$$

$$v_{fdisk} = 1.15\sqrt{g h_0}$$

calculated translational speed of the disk at the bottom of the incline

The experimental translational speed v_{edisk} of the disk at the bottom of the incline:

$$\bar{v}_{edisk} = \frac{d}{t_{averagedisk}}$$

$$\bar{v}_{edisk} = \frac{1}{2}(v_{0disk} + v_{edisk}) = \frac{1}{2} v_{edisk}$$

$$v_{edisk} = 2\bar{v}_{edisk}$$

$$v_{edisk} = \frac{2d}{t_{averagedisk}}$$

To determine the calculated final velocity v_{fring} for the ring:

$$\frac{1}{2} m v_{fring}^2 + \frac{1}{2} I \omega_{fring}^2 = m g h_0$$

$$\frac{1}{2} m v_{fring}^2 + \frac{1}{2} (m r^2) \omega_{fring}^2 = m g h_0$$

$$v_{fring}^2 + r^2 \omega_{fring}^2 = 2 g h_0$$

multiplying both sides by 2 and cancelling out *m*

$$v_{fring} = \omega_{fring} r$$

$$\omega_{fring} = \frac{v_{fring}}{r}$$

$$v_{fring}^2 + r^2 \left(\frac{v_{fring}}{r} \right)^2 = 2 g h_0$$

substitution

$$v_{fring}^2 + v_{fring}^2 = 2gh_0$$ simplify

$$2v_{fring}^2 = 2gh_0$$

$$v_{fring}^2 = gh_0$$

$$v_{fring} = \sqrt{gh_0}$$ calculated translational speed of the ring at the bottom of the incline

The experimental translational speed v_{ering} of the ring at the bottom of the incline:

$$\bar{v}_{ering} = \frac{d}{t_{averagering}}$$

$$\bar{v}_{ering} = \frac{1}{2}(v_{0ring} + v_{ering}) = \frac{1}{2}v_{ering}$$

$$v_{ering} = 2\bar{v}_{ering}$$

$$v_{ering} = \frac{2d}{t_{averagering}}$$

IV. Equipment and Materials

Moment of inertia disk and ring, two inclined planes constructed from white plastic rain gutters, stop watches, meter stick, calculator

V. Procedure

1. The distance *d* must be the same for both inclines, and as indicated .

2. Rest the upper ends of the inclined planes on the lab bench, desk, pile of books or some other objects. They must be parallel, aligned at both ends, at the same height at the upper ends. The tape marks should be near the upper ends and lined up with each other.

3. Measure the vertical height h_0 from the floor to the tape mark and record it in Table 1.

4. Release the disk and the ring and record how long it takes each of them to reach the bottom of the incline. One team member should record the time for the disk and one team member should record the time for the ring.

5. Run 2 more trials using the current distance *d* then run 12 additional trials, 3 each using the same distance *d* and height h_0. This will result in a total of 15 trials, 5 sets of 3 each at the same distance *d* and height h_0. Complete Table 1.

6. Complete Table 2.

VI. Data

Table 1 Disk and Ring Times

A	B	C	D	E	F	G	H
Set 1 incline data:		Distance along inclines in m:		d = 2.5 m	Height in m:		h_0 =
Time t_{disk} in s				Time t_{ring} in s			
Trial 1	Trial 2	Trial 3	Average time $t_{average\ disk}$	Trial 1	Trial 2	Trial 3	Average time $t_{average\ ring}$
Set 2 incline data:		Distance along inclines in m:		d = 2.0 m	Height in m:		h_0 =
Time t_{disk} in s				Time t_{ring} in s			
Trial 1	Trial 2	Trial 3	Average time $t_{average\ disk}$	Trial 1	Trial 2	Trial 3	Average time $t_{average\ ring}$
Set 3 incline data:		Distance along inclines in m:		d = 1.5 m	Height in m:		h_0 =
Time t_{disk} in s				Time t_{ring} in s			
Trial 1	Trial 2	Trial 3	Average time $t_{average\ disk}$	Trial 1	Trial 2	Trial 3	Average time $t_{average\ ring}$
Set 4 incline data:		Distance along inclines in m:		d = 1.0 m	Height in m:		h_0 =
Time t_{disk} in s				Time t_{ring} in s			
Trial 1	Trial 2	Trial 3	Average time $t_{average\ disk}$	Trial 1	Trial 2	Trial 3	Average time $t_{average\ ring}$
Set 5 incline data:		Distance along inclines in m:		d = 0.5 m	Height in m:		h_0 =
Trial 1	Trial 2	Trial 3	Average time $t_{average\ disk}$	Trial 1	Trial 2	Trial 3	Average time $t_{average\ ring}$

Table 2 Disk and Ring Speeds at the Bottom of the Incline

A	B	C	D = |B – C|	E	F	G = |E – F|
	Disk			Ring		
Set	Calculated translational speed v_{fdisk} in m/s	Experimental translational speed v_{edisk} in m/s	Difference between calculated and experimental translational speeds in m/s	Calculated translational speed v_{fring} in m/s	Experimental translational speed v_{ering} in m/s	Difference between calculated and experimental translational speeds in m/s
1						
2						
3						
4						
5						

VII. Discussion Questions

1. Which object, the disk or the ring, has greater translational speed at the bottom of the incline? Provide a mathematical explanation to support your answer.

2. Simplify the ratio $\frac{v_{fdisk}}{v_{fring}}$ as much as possible and find its numerical value.

3. What factors could explain the differences between the experimental and calculated velocities at the bottom of the incline?

4. Using the conditions listed in the beginning of the Calculations section, derive the velocity equation for a solid sphere with its axis through the center and simplify as much as possible.

5. Using the conditions listed in the beginning of the Calculations section, derive the velocity equation for a spherical shell with its axis through the center and simplify as much as possible.

6. Simplify the ratio $\frac{v_{fsphere}}{v_{fshell}}$ as much as possible and find its numerical value.

7. Compare the ratios $\frac{v_{fdisk}}{v_{fring}}$ and $\frac{v_{fsphere}}{v_{fshell}}$. What similarities are there between the velocities and the mass distributions for these two cases?