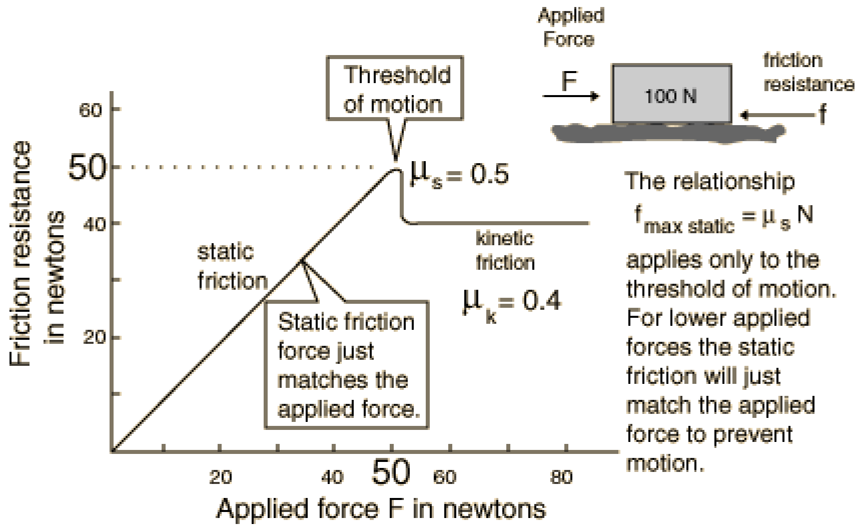
I. Objectives

1. Investigate the behavior of sliding and rolling masses.
2. Review the concepts of static and kinetic friction, velocity, acceleration, and force.

II. Introduction

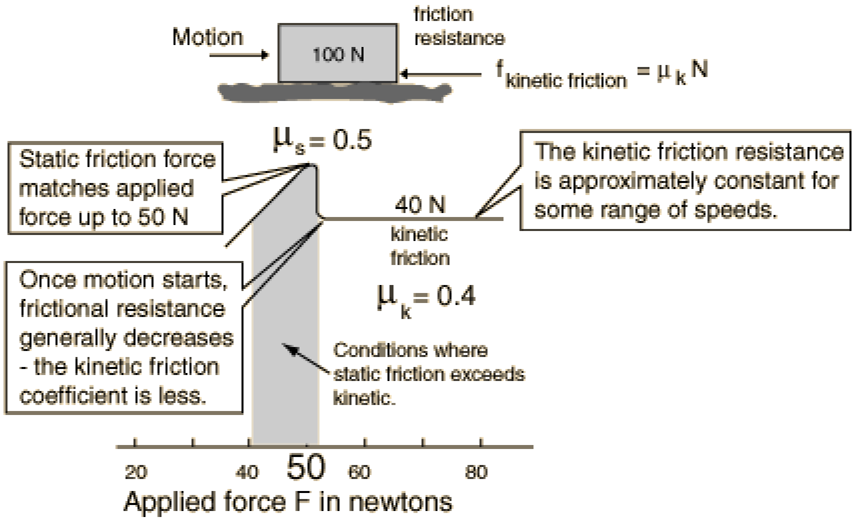
Static Friction



In making a distinction between static and kinetic coefficients of friction, we are dealing with an aspect of "real world" common experience with a phenomenon. The difference between static and kinetic coefficients obtained in simple experiments like wooden blocks sliding on wooden inclines roughly follows the model depicted in the friction plot from which the illustration above is taken. This difference may arise from irregularities, surface contaminants, etc. which defy precise description. When such experiments are carried out with smooth metal blocks which are carefully cleaned, the difference between static and kinetic coefficients tends to disappear. When coefficients of friction are quoted for specific surface combinations, it is the kinetic coefficient which is generally quoted since it is the more reliable number.

Static frictional forces from the interlocking of the irregularities of two surfaces will increase to prevent any relative motion up until some limit where motion occurs. It is that threshold of motion which is characterized by the coefficient of static friction. The coefficient of static friction is typically larger than the coefficient of kinetic friction. (http://hyperphysics.phy-astr.gsu.edu/hbase/frict2.html)

Kinetic Friction



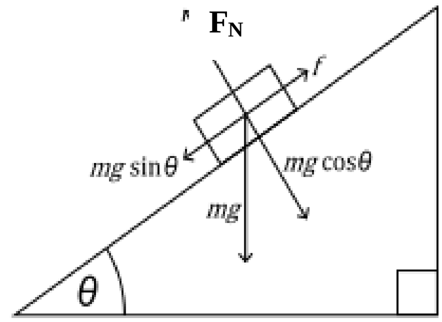
When two surfaces are moving with respect to one another, the frictional resistance is almost constant over a wide range of low speeds. The coefficient is typically less than the coefficient of static friction, reflecting the common experience that it is easier to keep something in motion across a horizontal surface than to start it in motion from rest. (http://hyperphysics.phy-astr.gsu.edu/hbase/frict2.html)

Acceleration of Gravity

Galileo thought a lot about the motion of falling objects, and was interested in the forces that acted upon objects in free-fall and whether those forces affect different types of objects differently. In Galileo's time, Aristotle's centuries-old assertion that heavier objects fall faster than lighter objects was almost universally accepted. Galileo acknowledged the difference in fall times for feathers and hammers, and conducted real-world experiments to test his ideas.

Even with a modern stopwatch, it is difficult to accurately measure an object's free-fall velocity or its rate of acceleration. Galileo lacked the timing equipment we have today, but he created an ingenious solution to slow down falling objects. He showed that a ball rolling down an incline would accelerate in the same way as a free-falling object, but more slowly. Using a straight, gently sloped piece of wood with a groove running down the center, an inclined plane, he was able to slow the effect of gravity on objects. His inclined plane allowed Galileo to accurately measure acceleration with simple instruments and ultimately to prove that, in the absence of other forces such as air resistance, gravity causes all falling objects to accelerate toward Earth at the same rate, and for the incline the acceleration is $g \sin \theta$.
(http://www.teachersdomain.org/resources/phy03/sci/phys/mfw/galileoplane/index.html)

III. Calculations



Stationary block: We also know that for a *stationary* block on the incline that the acceleration in the x-direction is 0 m/s^2 and that the magnitude of the static frictional force and the magnitude of the weight component must balance:

$$f_s = mg\sin\theta$$

$$\sum F_x = f_s - mg\sin\theta = 0\ N$$

(http://en.wikipedia.org/wiki/Inclined_plane)

$$f_s^{MAX} = \mu_s F_N = \mu_s mg\cos\theta$$ by definition

$$f_s^{MAX} \geq f_s$$ by definition

$$\mu_s mg\cos\theta \geq mg\sin\theta$$ substitution

$$\mu_s \cos\theta \geq \sin\theta$$ cancelling out mg

$$\mu_s \geq \frac{\sin\theta}{\cos\theta} \quad \text{for } \cos\theta > 0 \text{ and } 0 < \theta < 90^0$$

$$\mu_s \geq \tan\theta$$

Moving block: We know that for a *moving* block on the incline:

$$W = mg$$

$$F_N = W\cos\theta = mg\cos\theta$$

$$f_k = \mu_k F_N$$ by definition

The sum of the forces in the *x* direction along the incline:

$$\sum F_x = f_k - W\sin\theta = ma_x$$

$$\sum F_x = \mu_k F_N - mg\sin\theta = ma_x$$ substitution

$$\sum F_x = \mu_k mg\cos\theta - mg\sin\theta = ma_x$$ substitution

$$\mu_k g\cos\theta - g\sin\theta = a_x$$ cancelling m from both sides

$$a_x = g(\mu_k \cos\theta - \sin\theta)$$ factoring out g

The sum of the forces in the *y* direction for *both* a stationary and moving block:

$$\sum F_y = F_N - W\cos\theta = ma_y$$

$$\sum F_y = mg\cos\theta - mg\cos\theta = 0\ N$$

$$a_y = 0\ m/s^2$$

because the block is not falling through or flying off of the incline.

IV. Equipment and Materials

Calculator.

V. Procedure

1. Assume that the mass on the incline is in contact with the incline and is accelerating downward and to the left, per the diagram above, that positive x is to the right, parallel to the surface of the incline, and that positive y is upward, perpendicular to the incline. **Measure angles from the y-axis, and use *mg* for weight, not *W*.**

2. Your instructor will either provide each team with a value for the mass, the angle, and the coefficient of kinetic friction, per Table 1 below and/or will provide you with a Group # and data needed for Table 1.

3. Calculate the weight of the block and write the answers in Table 1. Remember that ***g* is negative**.

4. Complete Table 2. Write the correct equations required to calculate the values and the correct numerical results. You must include the correct equations, the correct signs for the equations, and the correct numerical results to receive credit. **If a quantity is 0, you must include it.** The final equation for the sum of the forces in the *x* direction must include **only** the variables μ_k, m, g, Θ, and a_x.

VI. Data

Table 1 Problem Parameters

A	B
parameter	numeric value
block mass *m* in kg	
calculated block weight *mg* in N	
angle Θ in degrees	
coefficient of kinetic friction μ_k	
calculated acceleration a_x in m/s^2	

Table 2 Force Equations and Numerical Results

A	B	C	D	E
Force	x-component equation	x-component value in N	y-component equation	y-component value in N
block weight *mg*				
F_N				
f_k				
$\sum F =$				

VII. Discussion Questions

1. Explain why we are using $\sin\theta$ in the equation for $\sum F_x$ and $\cos\theta$ in the equation for $\sum F_y$.

2. Explain how we know that $\sum F_y = 0\,\text{N}$ for the block in this lab.

3. What variable does acceleration a_x not depend on? Explain.

4. Calculate the acceleration a_x of the block and write the answer in the last row of Table 1. Explain why, in *this particular problem*, a_x is negative.

5. Now assume that you can change the angle θ. What is the *maximum* angle at which the incline could be set and at which the acceleration a_x would be 0 m/s^2? Assume that μ_k does not change. Show your work.