I. Objectives

1. Investigate the relationships among distance, time, velocity and acceleration.
2. Determine when two moving objects have traveled the same distance from an initial reference point using one-dimensional kinematics equations.
3. Practice solving for variables in kinematics equations.

II. Introduction

The goal of kinematics is to develop models to describe and explain the motion of real-world objects using words, diagrams, numbers, graphs, and equations. Using these models we can make predictions about the behavior of moving objects in relation to other objects to help us understand the laws of physics and apply them to real-world situations.

Linear kinematics usually begins by studying the linear motion of an object. We can graph the object's velocity using the *x*-axis to represent time, and the *y*-axis to represent distance, and its acceleration using the *x*-axis to represent time, and the *y*-axis to represent velocity.

III. Calculations

Variables:

v_0 = initial velocity in m/s

$v = v_f$ = final velocity in m/s

t = time in s

a = acceleration in m/s^2, assumed to be constant

x = distance in m

We will use some of the following linear kinematic equations, which will be referred to throughout the lab and the semester:

1. $v = v_0 + at$
2. $x = \frac{1}{2}(v_0 + v)t$
3. $x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2ax$

We'll use these to solve some real world problems.

You can assume all of the following:

- Your variables are designated with a subscript of 1 and those of your lab partner with a subscript of 2.
- You and your lab partner, for each of the trials, are 100.0 m apart and that you walk toward each other in order to complete today's lab.
- Your starting location 0.0 m and your lab partner's starting location is designated as 100.0 m. This also means that when you meet your distance is x_1 meters from the starting point and the distance traveled by your lab partner is $x_2 = 100.0 - x_1$ meters.
- Because we are calculating the magnitudes, not the directions of the velocities and accelerations we will use positive numbers for both in our data.
- When you meet, you are both the same distance from your starting point, and you are also the same distance from your lab partner's starting point, however, that does not imply that you meet at the middle of the distance between you.
- When you meet, you have each been walking for the same amount of time, meaning that $t = t_1 = t_2$.
- You each walk with constant, but not necessarily zero, acceleration.

For this example, assume that:

- your initial and final velocities are the same, $v_{01} = v_{f1} = 2.0$ m/s
- your lab partner's initial and final velocities are the same, $v_{02} = v_{f2} = 3.0$ m/s,
- neither of you is accelerating so that $a_1 = a_2 = 0.0$ m/s^2.

Based on this data we can set up the following equations, leaving out the units temporarily:

$$x_1 = 2.0t \quad \rightarrow \quad t = \frac{x_1}{2.0}$$

$$x_2 = 3.0t \quad \rightarrow \quad 100.0 - x_1 = 3.0t \quad \rightarrow \quad t = \frac{100.0 - x_1}{3.0}$$

$$\frac{x_1}{2.0} = \frac{100 - x_1}{3.0} \quad \rightarrow \quad 3.0x_1 = 200.0 - 2.0x_1 \quad \rightarrow \quad 5.0x_1 = 200.0 \quad \rightarrow \quad x_1 = 40.0$$

Using the equations for time t for each of you:

$$t = \frac{40.0}{2.0} = \frac{100.0 - 40.0}{3.0} = 20.0\ s$$

This means that you meet 40.0 m from your starting point of 0.0 m after 20.0 s. This method works well for simple problems where your accelerations are both 0.0 m/s^2, however, we need to develop a more general method.

Now assume that you and your lab partner each have non-zero initial velocities and non-zero accelerations, which provides a more general case for solving this kind of problem:

v_{01} = 1.8 m/s

v_{02} = 1.5 m/s

a_1 = 0.7 m/s^2

a_2 = 0.6 m/s^2

The kinematics equations for you:

$$x_1 = v_{01}t + \frac{1}{2}a_1t^2 \qquad x_1 = \frac{1}{2}a_1t^2 + v_{01}t$$

The kinematics equations for your lab partner:

$$x_2 = v_{02}t + \frac{1}{2}a_2t^2 \qquad x_2 = \frac{1}{2}a_2t^2 + v_{02}t$$

$$-100.0 + x_1 = -v_{02}t - \frac{1}{2}a_2t^2$$

$$x_1 = -\frac{1}{2}a_2t^2 - v_{02}t + 100.0$$

Setting the two equations for x_1 equal to each other and simplifying:

$$\frac{1}{2}a_1t^2 + v_{01}t = -\frac{1}{2}a_2t^2 - v_{02}t + 100.0$$

$$\frac{1}{2}(a_1 + a_2)t^2 + (v_{01} + v_{02})t - 100.0 = 0.0$$

Substituting the values given in the example and simplifying:

$$\frac{1}{2}(0.7 + 0.6)t^2 + (1.8 + 1.5)t - 100.0 = 0.0$$

$0.65t^2 + 3.3t - 100.0 = 0.0$

Then, using the quadratic formula we can calculate the possible values of t:

$At^2 + Bt + C = 0$

$$A = \frac{1}{2}(a_1 + a_2) = \frac{1}{2}(0.7 + 0.6) = 0.65$$

$B = v_{01} + v_{02} = 1.8 + 1.5 = 3.3$

$C = -100$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$t = \frac{-3.3 \pm \sqrt{(3.3)^2 - 4(0.65)(-100)}}{2(0.65)}$$

t = 10.1 s or t = -15.2 s, however, we disregard the negative solution of -15.2 s.

In this example, you and your lab partner meet in 10.1 s, and we can calculate your distance from your starting point using equation 3:

$x_1 = (1.8 \text{ m/s})(10.1 \text{ s}) + (0.5)(0.7 \text{ m/s}^2)(10.1 \text{ s})^2 = 53.9 \text{ m}$

And to check your lab partner's distance x_2:

$x_2 = (1.5 \text{ m/s})(10.1 \text{ s}) + (0.5)(0.6 \text{ m/s}^2)(10.1 \text{ s})^2 = 45.8 \text{ m}$

We know that $x_2 = 100.0 - x_1 = 100.0 - 53.9 \text{ m} = 46.1 \text{ m}$, very close to our answer for x_2.

We can derive the general equation for t using the example above:

$$t = \frac{-(v_{01} + v_{02}) \pm \sqrt{(v_{01} + v_{02})^2 - 4\left(\frac{1}{2}\right)(a_1 + a_2)(-100)}}{2\left(\frac{1}{2}\right)(a_1 + a_2)}$$

$$t = \frac{-(v_{01} + v_{02}) \pm \sqrt{(v_{01} + v_{02})^2 + (200)(a_1 + a_2)}}{a_1 + a_2}$$

If we were using a distance d instead of 100 meters we could rewrite this equation as:

$$t = \frac{-(v_{01} + v_{02}) \pm \sqrt{(v_{01} + v_{02})^2 - 4\left(\frac{1}{2}\right)(a_1 + a_2)(-d)}}{2\left(\frac{1}{2}\right)(a_1 + a_2)}$$

$$t = \frac{-(v_{01} + v_{02}) \pm \sqrt{(v_{01} + v_{02})^2 + (2d)(a_1 + a_2)}}{a_1 + a_2}$$

IV. Equipment and Materials

Calculator

V. Procedure

1. Complete Table 1 using kinematics equations and the provided data.

VI. Data

Table 1 Velocity, Acceleration, and Time

A	B	C	D	E	F	G	H
Trial	My initial velocity v_{01} in m/s	Lab partner's initial velocity v_{02} in m/s	My accel-eration a_1 in m/s^2	Lab partner's accel-eration a_2 in m/s^2	Time t until we meet in s	My final velocity v_{f1} in m/s calculate using equation 1	Lab partner's final velocity v_{f2} in m/s calculated using equation 1
1	1.00	1.00	0.01	0.01			
2	1.05	1.02	0.03	0.03			
3	1.10	1.05	0.02	0.02			
4	1.15	1.20	0.03	0.03			
5	1.20	1.40	0.04	0.04			

Table 2 Verifying Calculations

A	I	J	K	L	M
Trial	Distance x_1 in m from my starting point when we meet calculated using equation 3	Distance x_2 in m from lab partner's starting point when we meet calculated using equation 3	Distance x_2 in m from lab partner's starting point when we meet calculated using $100 - x_1$	Distance x_1 in m from my starting point when we meet calculated using equation 4	Distance x_2 in m from lab partner's starting point when we meet calculated using equation 4
1					
2					
3					
4					
5					

VII. Discussion Questions

1. Solve equation 1 for a

2. Solve equation 1 for t

3. Assuming that $v = v_0$, show the steps and simplify equation 2 as much as possible.

4. Solve equation 4 for v

5. Solve equation 4 for v_0

6. Solve equation 4 for a

7. Solve equation 4 for x

8. What would the value for C be if the distance between you was 250 meters instead of 100 meters?