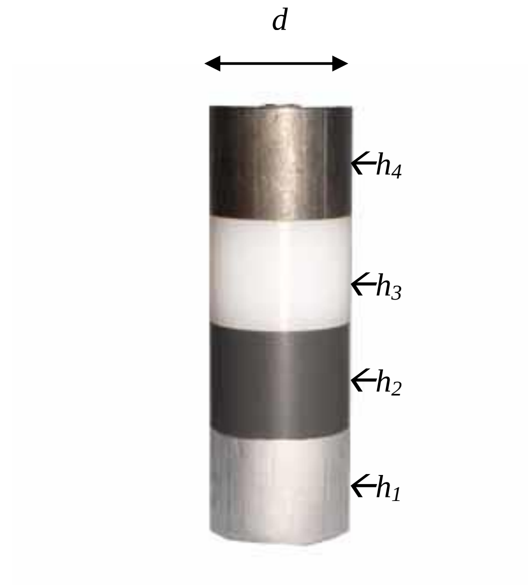
I. Objectives

1. Calculate the center of gravity for various configurations of cylinders of different masses, but nearly identical volumes.
2. Determine the angle at which the configuration falls.

II. Introduction



There are many leaning towers, however, the most famous is the Leaning Tower of Pisa in Italy, which slants 5.5^0 from vertical. It is 55.86 m from the ground on the lowest side and 56.70 m on the highest side. (Source:

http://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa)

III. Calculations

The center of gravity is calculated using the equation:

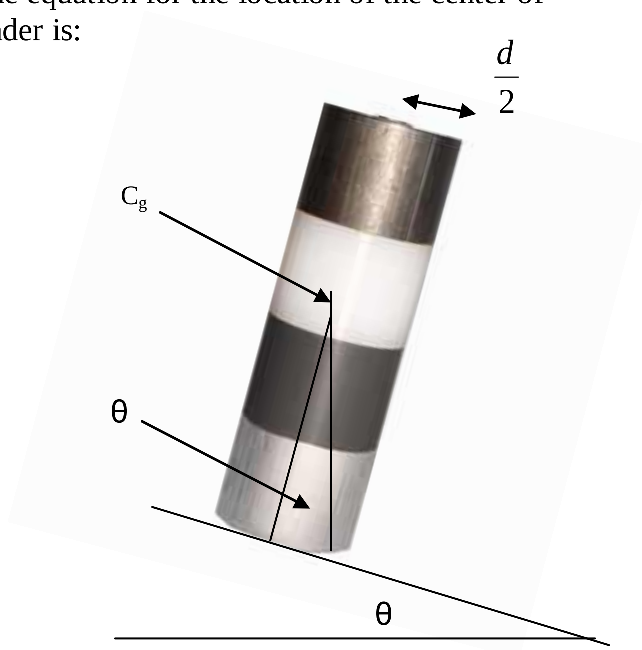
$$c_g = \frac{m_1 h_1 + m_2 h_2 + m_3 h_3 + m_4 h_4 + m_r h_r}{m_1 + m_2 + m_3 + m_4 + m_r}$$

where m_1, m_2, m_3, m_4, and m_r represent the cylinder masses m_a, m_b, m_c, and m_d, and the rod mass m_r, as listed in Table 1, depending on the cylinder configuration listed in Table 3.

For example, if the brass cylinder is on the bottom, followed by the aluminum cylinder, the PVC cylinder, and the polyethylene cylinder at the top, the equation for the location of the center of gravity from the base of the combined untilted cylinder is:

$$c_g = \frac{m_b h_1 + m_a h_2 + m_d h_3 + m_c h_4 + m_r h_r}{m_a + m_b + m_c + m_d + m_r}$$

The cylinder should fall when tilted at an angle θ:



$$\tan\theta = \frac{\left(\frac{1}{2}d\right)}{c_g} = \frac{d}{2c_g}$$

$$\theta = \arctan\left(\frac{d}{2c_g}\right)$$

III. Equipment and Materials

Four cylinder Leaning Tower of Pisa apparatus, electronic balance, inclined plane, rubber mat, level, metric ruler, calculator

IV. Procedure

1. Measure and record the length of the rod. Rod length = _____ cm.

2. The height of the center of gravity of the rod is half its length. Rod center of gravity h_r = _____ cm.

3. Measure and record the diameter d of any one of the cylinders. They should all be approximately the same diameter. Cylinder diameter d = _____ cm.

4. Record the masses of each of the four Leaning Tower of Pisa cylinders and the rod in Table 1.

5. Measure and record the height of any one of the cylinders. They should all be approximately the same height. Cylinder height h = _____ cm. Now divide the height by two to determine h_1, the height of the center of gravity of the first (bottom) cylinder.

6. Calculate and record the heights of the center of gravity of each of the remaining cylinders, h_2, h_3, and h_4, by adding the **total** height h, **not** half the height, to each succeeding height. Remember that these heights will remain the same throughout the lab for all trials.

7. Place the inclined plane flat on the table or lab bench and be sure that it is level.

8. Place the rubber mat near the bottom of the inclined plane. It will prevent the cylinder from sliding off the plane.

9. Set up the first configuration for the first trial in Table 3. Calculate the center of gravity and the fall angle. **Be sure to use the correct masses for the appropriate mass variables in the center of gravity equation. Remember that the order in which they are used in each calculation is dependant on the cylinder configuration, and don't forget to include $m_r h_r$ as part of the sum in the numerator of each calculation.**

10. Increase the angle of the inclined plane until the Leaning Tower falls over. Record this actual fall angle.

11. Calculate the difference between the calculated and actual fall angles.

12. Complete the process for the remaining trials listed in Table 3.

V. Data

Table 1 Masses

A	B	C
Cylinder material	Cylinder designation	Mass in grams
aluminum (silver colored metal)	a	$m_a =$
brass (gold colored metal)	b	$m_b =$
polyethylene (white plastic)	c	$m_c =$
PVC (tan plastic)	d	$m_d =$
rod	r	$m_r =$
Total mass	$m_a + m_b + m_c + m_d + m_r =$	

Table 2 Cylinder Center of Gravity Heights

A	B
Height designations (first number = bottom cylinder, last number = top cylinder)	Cylinder center of gravity height in cm
$h_1 = \frac{1}{2}h =$	
$h_2 = \frac{3}{2}h =$	
$h_3 = \frac{5}{2}h =$	
$h_4 = \frac{7}{2}h =$	

Table 3 Cylinder Configurations, Centers of Gravity, and Fall Angle

A	B	C	D	E	F = | D – E |
Trial	Cylinder configuration (first letter = bottom cylinder, last letter = top cylinder)	Calculated height of the center of gravity c_g in cm	Calculated fall angle θ in degrees	Actual fall angle θ in degrees	Difference between calculated and actual fall angle θ in degrees
1	a, b, c, d				
2	a, b, d, c				
3	a, c, b, d				
4	a, c, d, b				
5	a, d, b, c				
6	a, d, c, b				
7	b, a, c, d				
8	b, a, d, c				
9	b, c, a, d				
10	b, c, d, a				
11	b, d, a, c				
12	b, d, c, a				
13	c, a, b, d				
14	c, a, d, b				
15	c, b, a, d				
16	c, b, d, a				
17	c, d, a, b				
18	c, d, b, a				
19	d, a, b, c				
20	d, a, c, b				
21	d, b, a, c				
22	d, b, c, a				
23	d, c, a, b				
24	d, c, b, a				

VI. Discussion Questions

1. Under what conditions and/or configurations was the center of gravity nearly the same? Why?

2. Compare the calculated and actual fall angles. Were they nearly the same or different? What characteristics of the experimental setup could explain the differences? Why?