I. Objectives

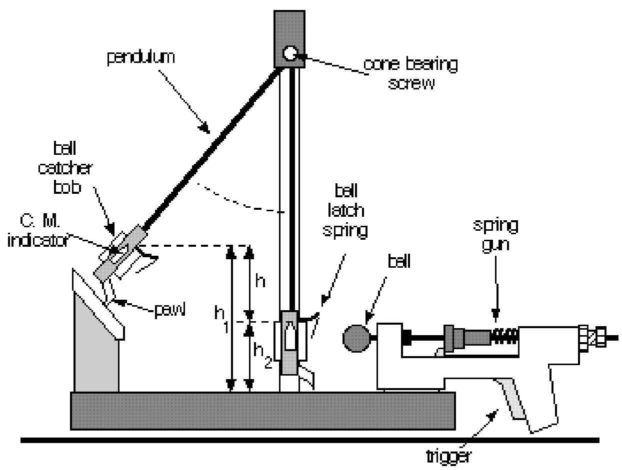
1. Apply kinematics equations and conservation laws.
2. Calculate the horizontal velocity of a ball after it is fired from a ballistic pendulum.

II. Introduction

This lab involves projectiles. You must wear appropriate safety gear.

The ballistic pendulum is a classic example of a dissipative collision in which conservation of momentum can be used for analysis, but conservation of energy during the collision cannot be invoked because some of the energy is lost.

After the collision, however, the metal ball and the ballistic pendulum "stick" together and conservation of energy can be applied in the swing of the combined masses upward, since the gravitational potential energy is conservative.



III. Calculations

Part 1 Projectile

The laws of kinematics tell us:

$$x = x_0 + v_{0x}t + \frac{1}{2}at^2$$, the distance the ball travels in the *x* direction

$$y = y_0 + v_{0y}t + \frac{1}{2}gt^2$$, the distance the ball travels in the *y* direction

Placing the origin of our coordinate system at the point of release of the ball,

$$x_0 = y_0 = 0\ m$$

and our equations simplify to:

$$x = v_{0x}t$$

$$y = v_{0y}t + \frac{1}{2}gt^2$$

Since the ball is fired only with an initial horizontal velocity v_{0x}:

$$v_{before} = \bar{v}_{0x}$$

$$v_{oy} = 0 \text{ m/s}$$

$$y = \frac{1}{2}gt^2$$

Solving for *t*:

$$t = \frac{x}{v_{ox}}$$

and substituting this result in the equation for *y*:

$$y = \frac{1}{2}g\left(\frac{x}{v_{ox}}\right)^2 = \frac{gx^2}{2v_{0x}^2}$$

$$v_{ox}^2 = \frac{gx^2}{2y}$$

$$v_{ox} = \sqrt{\frac{gx^2}{2y}}$$

$$v_{before} = \sqrt{\frac{gx^2}{2y}}$$

$$v_{before} = v_{0x}$$

We can use this formula to compute the initial velocity of the metal ball.

Part 2 Ballistic Pendulum

We need to define a few symbols to simplify the remaining calculations:

m = mass of metal ball

M = mass of the ballistic pendulum arm

v_{after} = velocity of the combined metal ball and pendulum arm

h = height of the ballistic pendulum arm after impact with the metal ball

After the metal ball and ballistic pendulum collide they move as one body. This type of collision is totally inelastic and kinetic energy is not conserved. If no net horizontal forces act during the collision, the momentum of the metal ball before the collision equals the momentum of the combined metal ball and pendulum arm after the collision:

$$mv_{before} = (m+M)v_{after}$$

The kinetic energy of the metal ball is calculated from:

$$KE_{before} = \frac{1}{2}mv_{before}^2$$

$$v_{after} = \frac{mv_{before}}{(m+M)}$$

We can equate the gravitational potential energy when the center of mass has risen to a height h and the system is at rest with the kinetic energy of the pendulum arm and metal ball just after the collision:

$$PE_{after} = KE_{after}$$

$$PE_{after} = (m+M)gh$$

$$KE_{after} = \frac{1}{2}(m+M)v_{after}^2$$

$$(m+M)gh = \frac{1}{2}(m+M)v_{after}^2$$

$$v_{after}^2 = 2gh$$

$$v_{after} = \sqrt{2gh}$$

Substituting for v_{after}:

$$(m+M)gh = \frac{1}{2}(m+M)\left(\frac{mv_{before}}{m+M}\right)^2$$

$$gh = \frac{1}{2}\left(\frac{mv_{before}}{m+M}\right)^2 \qquad \text{cancelling } m+M$$

$$v_{before}^2 = \frac{2gh(m+M)^2}{m^2}$$

$$v_{before} = \frac{(m+M)\sqrt{2gh}}{m}$$

$$v_{before} = \frac{(m+M)v_{after}}{m}$$

The ratio of the velocities is:

$$\frac{v_{after}}{v_{before}} = \frac{\sqrt{2gh}}{\frac{(m+M)\sqrt{2gh}}{m}}$$

$$\frac{v_{after}}{v_{before}} = \frac{m}{m+M} \qquad \text{cancelling } \sqrt{2gh}$$

We also need to calculate the height h of the ballistic pendulum arm after the impact:

$$h = L - L\cos\theta$$

where L is the length of the ballistic pendulum arm in meters, and θ is the change in the angle of the arm before and after the impact.

The ratio of the kinetic energies is:

$$\frac{KE_{after}}{KE_{before}} = \frac{\frac{1}{2}(m+M)v_{after}^2}{\frac{1}{2}mv_{before}^2}$$

$$\frac{KE_{after}}{KE_{before}} = \frac{(m+M)v_{after}^2}{mv_{before}^2}$$

$$\frac{KE_{after}}{KE_{before}} = \frac{(m+M)(2gh)}{m\left(\frac{(m+M)^2(2gh)}{m^2}\right)}$$

$$\frac{KE_{after}}{KE_{before}} = \frac{m^2(m+M)}{m(m+M)^2}$$

$$\frac{KE_{after}}{KE_{before}} = \frac{m}{m+M}$$

IV. Equipment and Materials

Safety goggles, ballistic pendulum, metal ball, sand container, electronic balance, meter stick, calculator

V. Procedure

Part 1 Projectile

1. **Note that you will use the average velocities in your calculations for the Discussion Questions from the last rows of Tables 1 and 2.**

2. **Do not stand in front of or look into the ballistic pendulum, whether it is loaded or not. Before firing the pendulum notify everyone in the area to move away from the metal ball path.** The pendulum has three different settings, be sure to use the same one throughout the experiment.

3. Measure the height *y* of the pendulum bob above the floor.

4. Move the pendulum arm out of the way, load the pendulum and fire the metal ball. Watch where the ball lands, retrieve it, then place the middle of the sand container at that location.

5. Reload the metal ball in the pendulum, and fire it for 5 trials. Measure and record the *x* distance for each trial.

6. Calculate v_{0x} using the equation $v_{ox} = \sqrt{\dfrac{gx^2}{2y}}$.

7. Complete Table 1

<u>Part 2 Ballistic Pendulum</u>

8. Measure the mass *m* of the metal ball, the mass *M* and length *L* of the ballistic pendulum arm.

9. Reload the ballistic pendulum and move the pendulum back into place. Be sure that the pendulum arm is vertical. Remember the location of the pendulum may not be at the 0^0 mark.

10. Fire the metal ball and record the change in angle θ.

11. Repeat this process at least four more times.

12. Calculate *h* using the equation $h = L - L\cos\theta$.

13. Calculate v_{before} using the equation $v_{before} = \dfrac{(m+M)\sqrt{2gh}}{m}$.

14. Complete Table 2.

VI. Data

Table 1 Projectile

A	B	C	D
Trial	Measured distance *y* above floor in m	Measured distance *x* in m	Calculated velocity v_{0x} of metal ball in m/s
1			
2			
3			
4			
5			
		Average:	$v_{before} = v_{0x} =$

Table 2 Ballistic Pendulum

A	B	C	D	E	F	G	H
Trial	Measured mass *m* of metal ball in kg	Measured mass *M* of ballistic pendulum arm in kg	Measured length *L* of ballistic pendulum arm in m	Experimental θ, change in angle of the ballistic pendulum arm in degrees	Calculated height *h* of ballistic pendulum arm in m	Calculated velocity v_{before} of the metal ball in m/s	Calculated velocity v_{after} of the metal ball in m/s
1							
2							
3							
4							
5							
					Averages:	$v_{before} =$	$v_{after} =$

VII. Discussion Questions

1. Explain why kinetic energy is not conserved after the metal ball impacts the ballistic pendulum.

2. Compare the numerical values of v_{before}, the average of the v_{0x} velocities from Table 1 and v_{before} in Table 2. Are they similar? What could account for any differences?

3. What is the algebraic relationship between the ratios of the kinetic energy, $\frac{KE_{after}}{KE_{before}}$ and the ratio of the velocities, $\frac{v_{after}}{v_{before}}$?

4. What is the equation for KE_{before}? Calculate the experimental kinetic energy KE_{before}, just before the impact, when only the mass of the metal ball is moving, using v_{before} from Table 2. Include your calculations.

5. What is the equation for KE_{after}? Calculate the experimental kinetic energy KE_{after}, just after the impact, when both the metal ball and the ballistic pendulum arm are moving as a single mass, using v_{after}. Include your calculations.

6. Calculate the experimental numeric value of $\frac{KE_{after}}{KE_{before}}$ using the results from the two previous questions.

7. Calculate the theoretical numeric value of $\frac{KE_{after}}{KE_{before}}$ using the equation $\frac{KE_{after}}{KE_{before}} = \frac{m}{m+M}$. Include your calculations.

8. Calculate the experimental numeric value of $\frac{v_{after}}{v_{before}}$ using the averages from Table 2 for v_{before} and v_{after}. Include your calculations.

9. Is the theoretical value of $\frac{KE_{after}}{KE_{before}}$ equal to the experimental values of $\frac{KE_{after}}{KE_{before}}$ and $\frac{v_{after}}{v_{before}}$? Why?