I. Objectives

1. Use the small angle formula and one of your calibrated angular measuring devices to calculate the diameter of the Moon.
2. Use the small angle formula to calculate the diameter of the Earth and the apparent maximum diameter of each of the planets as seen from the Sun and the Earth.

II. Introduction

Recall that a circle has 360^o and the circumference of a circle is C = 2 π d, where π = 3.14 and *d* = the radius of the circle. If we want to measure a piece of the circumference, also called an arc of a circle, D, we can utilize the small angle formula, derived below to measure the angle subtended by arc D.

III. Theory and Calculations

We need to use a little algebra and the small angle formula. You need not be concerned with all of the details of the algebra.

1. This formula simply sets up a ratio, where the length of arc D to the circumference $2\pi d$, is proportional to the angle θ, the angular diameter of an object, to the entire circle of 360^0:

$$\frac{D}{2\pi d} = \frac{\theta}{360^0}$$

2. By multiplying both sides of step 1 by 2 π d:

$$D = \frac{2\pi d\theta}{360^0}$$

3. $$\frac{2\pi}{360^0} = \frac{6.28}{360^0} = \frac{1}{57.296^0}$$

4. Convert from decimal degrees to arcseconds:

$$\frac{1}{57.296^0} x \frac{1^o}{3{,}600''} = \frac{1}{206{,}265''}$$

Note: remember that the symbol for arcsecond is ″

5. Small Angle Diameter formula:

$$D = \frac{d\,\theta}{206{,}265''}$$

from step 2, replacing $\frac{2\pi}{360^0}$ with $\frac{1}{206{,}265''}$

6. Small Angle Distance formula:

$$d = \frac{206{,}265'' \times D}{\theta}$$

by multiplying both sides of step 5 by 206,265" and dividing both sides of step 5 by θ.

7. Small Angle formula:

$$\theta = \frac{206{,}265'' \times D}{d}$$

by multiplying both sides of step 6 by θ and dividing both sides by *d*.

IV. Prelab Definitions

1. circumference

2. radius

3. arc

4. angle

5. latitude

6. inferior planet

7. superior planet

V. Lab Procedure

1. Use one of your calibrated angular measuring devices to determine θ, the angular diameter of the Moon. Because the angular diameter of the Moon is small you will need to be as accurate in your measurement as possible. You will also need to find the actual distance, d, to the Moon in kilometers, and the true diameter of the Moon. Calculate D, the estimated diameter of the Moon in kilometers using the Small Angle Diameter formula in step 5 of the Theory and Calculations section, above. Be sure that your answer makes sense! If the angular diameter that you measure is more than one degree, you need to redo your measurement.

Moon Data

A	actual distance d to the Moon in km	
B	angular diameter θ in degrees	
C	angular diameter θ in arcseconds C = B *x* 3600	
D	$D = \dfrac{d\,\theta}{206{,}265''} = \dfrac{A \times C}{206{,}265''}$	
E	true diameter of the Moon in km	
F	difference F = | D - E | in km	

2. In the *Angular Diameters* table below, calculate the diameter, D, of each solar system body by multiplying the radius of each, r_p, in column B by 2. Write the results in column C. Remember that the diameter, D, of a planet is twice its radius, $2r_p$. C = 2 x B.

3. Using the diameters, D, in column C and distances from the Sun, d, in column E, calculate the angular diameter, θ, in arcseconds of the planets as seen from the Sun using the Small Angle formula in step 7 of the Theory and Calculations section, above.

$$F = \frac{206{,}265'' \times C}{E}.$$

4. Calculate each solar system body's minimum distance from the Earth. Remember that this minimum distance is the distance between the Earth and planet when the Earth and planet are aligned on the same side of the Sun. For the Sun, this distance is simply the distance from the Earth to the Sun and d_{Earth} = distance of Earth from the Sun and d_{planet} = distance of planet from the Sun.

For inferior planets, this distance will be:

Minimum distance, d from Earth, km = d_{Earth} - d_{planet}, where G = 149,597,900 - E.

For superior planets, this distance will be:

Minimum distance, d from Earth, km = d_{planet} - d_{Earth}, G = E - 149,597,900.

5. Using the diameters in column C and minimum distances from Earth in column G, calculate the maximum angular diameter of the Sun and each of the planets as seen from the Earth using the formula in step 7 of the Theory and Calculations section, above.

$$H = \frac{206{,}265'' \times C}{G}$$

Angular Diameters

A	B	C	E	F	G	H
Solar system body	r_p in km	Diameter D in km	Distance d from Sun in km	θ in " as seen from Sun in degrees	Minimum distance d from Earth in km	Maximum θ in " as seen from Earth in degrees
Sun	695,990		N/A	N/A		
Mercury	2,439		57,900,000			
Venus	6,052		108,200,000			
Earth	6,378		149,597,900		N/A	N/A
Mars	3,398		227,900,000			
Jupiter	71,494		778,300,000			
Saturn	60,330		1,427,000,000			
Uranus	25,559		2,869,000,000			
Neptune	24,750		4,497,100,000			

VI. Lab Discussion

1. How close was the diameter you calculated to the true diameter of the Moon? Other than mathematical error, what are the possible sources of error?

2. Excluding measurement errors, for what scientific reasons might planets appear larger or smaller that the angular diameters calculated?

3. When the full Moon is close to the horizon it appears larger than when it is higher in the sky. Is it really larger? Explain your answer.

4. Explain why this method can't be used to measure the distances to the planets or to the stars.

5. Explain why we use the minimum distance of each planet from Earth in order to calculate its maximum angular diameter as seen from Earth.

6. Which planet has the maximum angular diameter as seen from the Sun?

7. Which planet has the maximum angular diameter as seen from the Earth?

8. Explain why the answers to questions 6. and 7. are different.