I. Objective

1. Duplicate Eratosthenes' 247 BC estimation of Earth's circumference

II. Introduction

One of the great feats of reason for its time was the estimation of Earth's circumference by Eratosthenes of Alexandria in 247 BC. If a person is on the surface of a nearly spherical planet and has a distant reference point, such as the Sun, then basic geometry can be applied to calculate the approximate circumference of the planet.

Historical note: The place where this was first demonstrated, Alexandria, Egypt, was founded by Alexander the Great in 331 BC. It became the largest city on Earth during that period and hosted the Library at Alexandria, home to the greatest scholars of the time. Two of the more well known Egyptian obelisks in Alexandria were Cleopatra's Needle and the Obelisk of Theodosius. However, neither of these was standing in 247 BC. The first arrived in 12 BC and the second did not arrive until 357 AD. The ancient Egyptians were prolific builders of monuments so some kind of obelisk was surely present in Eratosthenes' time. The lighthouse of Alexandria may have been completed around 247 BC, but its location next to the sea would not have been that conducive to measuring shadows nor was it proximate to the Library.

III. Materials

Tape measure (~25 m), protractor or height gauge

IV. Theory & Calculations

Eratosthenes determined that the sun shone to the bottom of a well at Syene on June 21, the date of the summer solstice. This meant that he made his observation at a latitude of 23.5^o N, when the Sun was directly overhead. He also happened to be monitoring the solar altitude angle θ in Alexandria, which he calculated by taking the ratio of the height h of an obelisk to the length d of the obelisk's shadow:

$$\tan \theta = h/d$$

On June 21, he measured a solar angle of 82.8^o, and knew that the distance between Alexandria and Syene was approximately 5,000 stadia, a Greek unit of measure approximately equal to 185 m.

Christopherson (2010) states, "Eratosthenes knew from geometry that the distance on the ground between Alexandria and Syene formed an arc of Earth's circumference equal to the complement of the solar angle at Alexandria ($\varphi = 90 - 82.8 = 7.2$). Since 7.2^o is roughly 1/50 of the 360^o in

Earth's total circumference, he knew that the distance on the surface must represent 1/50 of the Earth's polar circumference."

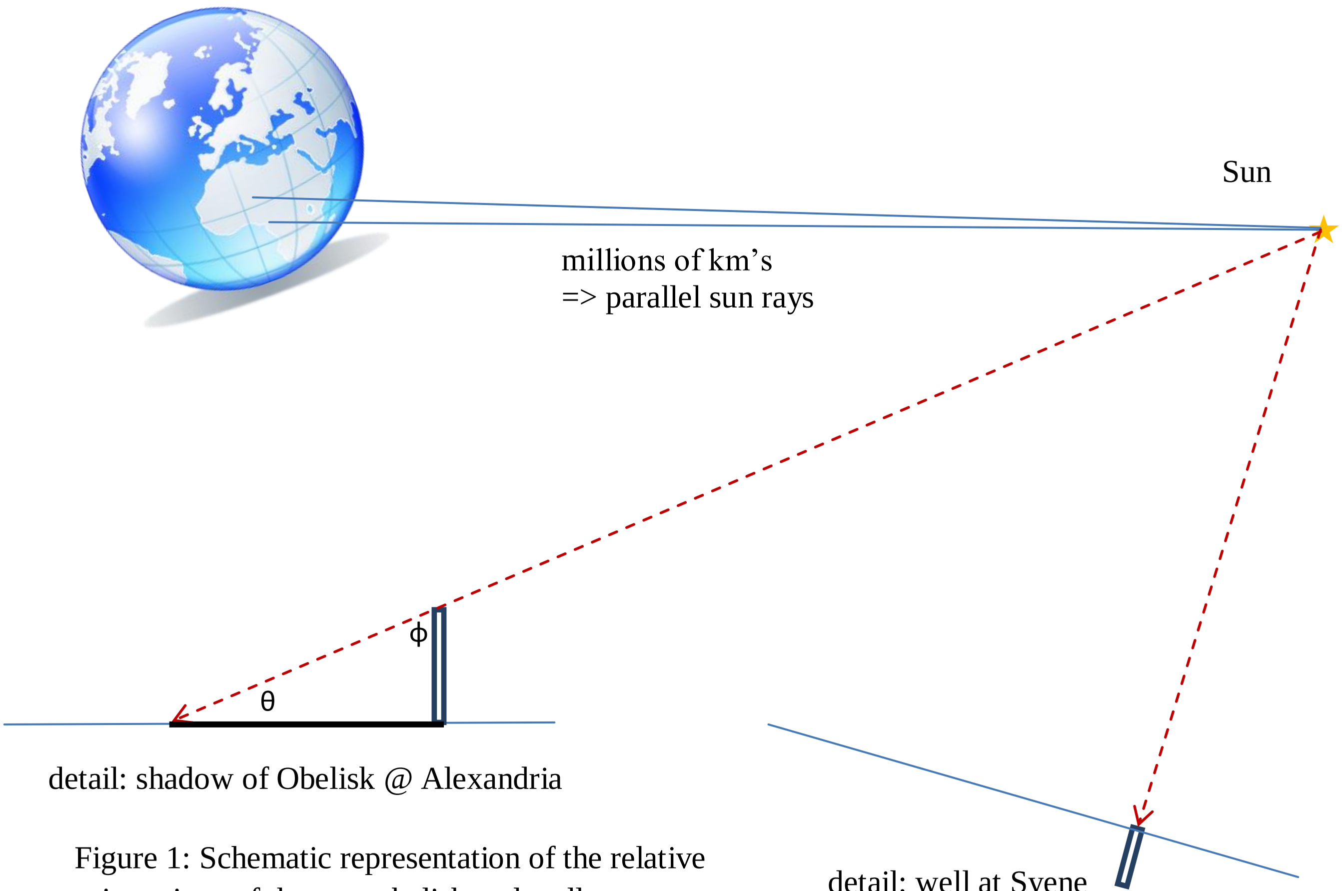


Figure 1: Schematic representation of the relative orientations of the sun, obelisk and well.

Also see Figure 1.12 in Christopherson (8^{th} ed).

We know:

1. Solar angle at two points, one of which happens to be directly overhead

$$\tan\theta = \frac{opp}{adj} = \frac{h}{d}, \ \theta = 82.8^{o}$$

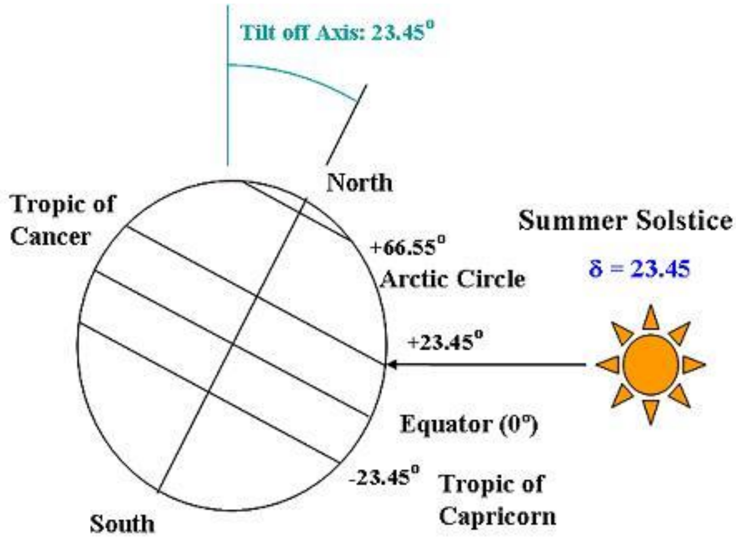
2. Distance between Syene and Alexandria, L = 5,000 stadia = 925 km (or from Google Earth)

3. From the geometry of a circle, the following ratios are equal:

$$\frac{90 - 82.8}{360} = \frac{5{,}000}{Earth's\ circumference}$$

thus Earth's circumference = 250,000 stadia or 46,250 km (true value = 40,008 km)

Our solar simulator provides us with the solar declination, the latitude of the sub-solar point.



Declination (°)	*Approximate dates*
+23.45	June 22
+20	May 21, July 24
+15	May 1, Aug 12
+10	Apr 16, Aug 28
+5	Apr 3, Sept 10
0	Mar 21, Sept 23
−5	Mar 8, Oct 6
−10	Feb 23, Oct 20
−15	Feb 9, Nov 3
−20	Jan 21, Nov 22
−23.45	Dec 22

Figure 2: Summer Solstice (www.geog.ucsb.edu/ideas/Insolation.html)

The angle between our latitude and the solar declination is the "included angle" φ, which is also the complement of the solar altitude angle obtained by measuring the shadow of the flagpole or obelisk, whatever is more convenient.

Finally, the height of the flagpole is another complication, but can be estimated by measuring the distance you are away from the flagpole while sighting along your arm to the top of the flagpole with your arm at 45°. Add to this the distance from the ground to your eye.

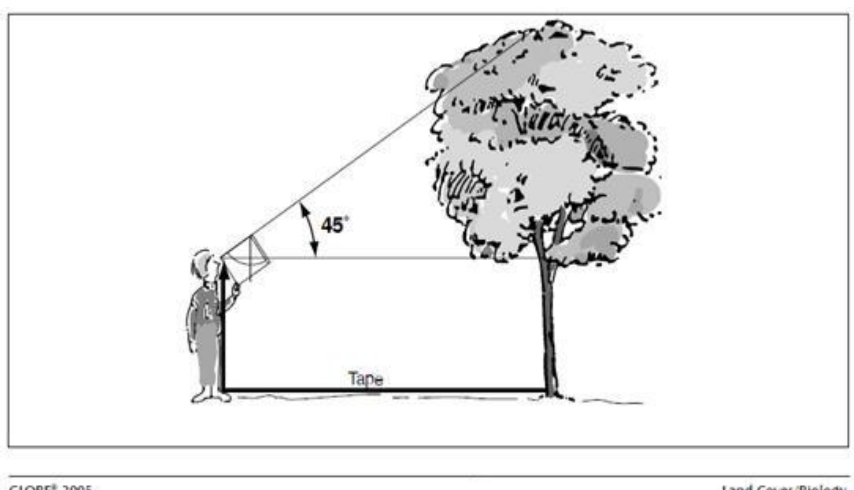


Figure 3: Estimating tree height, GLOBE Teacher Guide (2005)

V. Prelab Definitions

1. solar angle

2. solar declination

3. included angle

4. sub-solar point

5. obelisk

6. Tropic of Cancer

7. Tropic of Capricorn

VI. Lab Procedure

We are going to duplicate, to some degree, Eratosthenes' experiment by measuring the noon-time shadow cast by the flag pole and using a web-based solar calculator to locate where the sun is at zenith on both the first day of class in the Spring (approximately January 17) and the first day of class in the Fall (approximately August 25). The software will also allow us to estimate the solar angle for any other date. We can use Google Earth to measure the surface distance between our location and the sub-solar or nadir point. Then we will estimate the circumference of the Earth using the simple ratio equation shown above.

Complete Table 1 using the resources listed above, Google Earth, and noon observations of a flagpole's shadow. You will measure solar altitude for the current date and look up or calculate the remaining data.

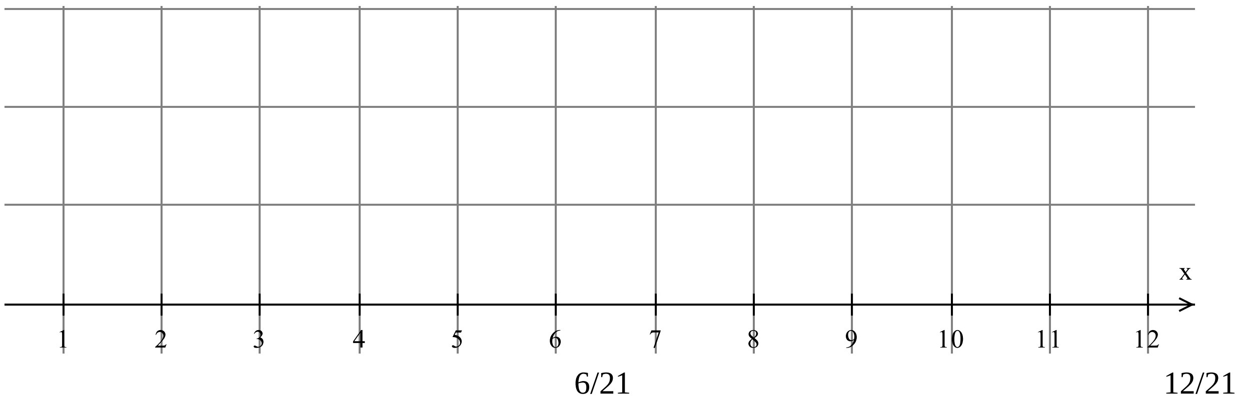
Table 1: Observational and Tabulated Data Used to Determine Earth's Circumference

Location	Alexandria	Tucson	Tucson
Observation date	June 21	Jan 17	Aug 30
Latitude	31.1	32.2	32.2
Solar declination (sub-solar point)	23.4		
Approx. distance	5000 st. = 925 km	km	km
Right angle	90.0	90.0	90.0
Included angle	7.2		
Solar altitude	82.8		
Circumference	250,000 st = 46,250 km	km	km

VII. Lab Discussion

1. Why might Eratosthenes have been monitoring the shadow of the obelisk at Alexandria?

2. Draw a graph of day of the year vs. shadow length for a location on the Tropic of Cancer. Be sure to label the dates of the solstices and equinoxes, which occur on 12/21, 3/21, 6/21, 9/21.



3. How close were your values to the true polar circumference of ~40,000 km?

4. Describe how each of the following could contribute to errors in the analysis:
   a. flagpole measurement

   b. solar simulation software

   c. Google Earth

5. How close was your measured solar altitude to the value calculated using the software?

6. In this lab are you measuring Earth's polar or equatorial circumference? Explain.

Lab courtesy of Dr. Jim Washburne