Harmonic Wave Equation Derivations Using Calculus

1) $\theta = \omega t$ — θ = angular displacement in rad, ω = angular velocity in rad/s

2) $\omega = \frac{2\pi}{T}$ — T = period in s

3) $f = \frac{1}{T}$ — f = frequency in cycles/s or Hz

4) $T = \frac{1}{f}$

5) $\omega = \frac{2\pi}{\frac{1}{f}} = 2\pi f$

6) $x = A\cos\theta = A\cos\omega t$ — x = linear displacement, A = amplitude

7) $v = \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d(A\cos\omega t)}{dt} = -A\omega\sin\omega t$

8) $v_{max} = A\omega$ — this occurs when $\sin\omega t = -1$

9) $a = \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d(-A\omega\sin\omega t)}{dt} = -A\omega^2\cos\omega t$

10) $a_{max} = A\omega^2$ — this occurs when $\cos\omega t = -1$

11) $F = -kx = ma$ — k = spring constant

12) $-k(A\cos\omega t) = m(-A\omega^2\cos\omega t)$ — substitution of x and a

13) $k = m\omega^2$ — cancellation of A, $\cos\omega t$, and the negative signs

14) $\omega = \sqrt{\frac{k}{m}}$

15) $\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$