PHY 121IN Physics of Yoyos

You may work with a partner during this activity. Although many tricks can be done with a yo-yo, it primarily moves up and down on a string, and its motion can be expressed by the following equations, where *T* is the tension in the string, r_a is the radius of the axle and *m* is the mass of the disks plus the mass of the axle, $m = 2m_d + m_a$:

$$\sum F_y = T - mg = ma_y \qquad \alpha = \frac{a_y}{r_a} \qquad Tr_a = I\alpha$$

1. Write the equation for *T* in terms of *I*, a_y and r_a.

$$Tr_a = \frac{Ia_y}{r_a} \rightarrow T = \frac{Ia_y}{r_a^2}$$

2. Eliminate *T* from the first equation above in the problem description using substitution and write the result.

$$\frac{Ia_y}{r_a^2} - mg = ma_y$$

3. Now solve this equation for a_y in terms of *I*, r_a, *m*, and *g*.

$$\frac{Ia_y}{r_a^2} - ma_y = mg \rightarrow a_y\left(\frac{I}{r_a^2} - m\right) = mg \rightarrow a_y = \frac{mg}{\frac{I}{r_a^2} - m} \rightarrow$$

$$a_y = \frac{mg}{\frac{I}{r_a^2} - \frac{mr_a^2}{r_a^2}} \rightarrow a_y = \frac{mg}{\frac{I - mr_a^2}{r_a^2}} \rightarrow a_y = \frac{mgr_a^2}{I - mr_a^2}$$

4. How would increasing the moment of inertia of the yo-yo affect its linear acceleration?

Increasing the moment of inertia decreases the linear acceleration because the denominator of the equation in step 3 increases.

5. We need to treat the yo-yo as a system consisting of **two** disks and an axle. Write the equation for the moment of inertia *I* of the yo-yo using m_d and r_d for the mass and radius of each disk, and m_a and r_a for the mass and radius of the axle, which you can assume is a very small disk. Simplify as much as possible.

$$I = 2\left(\frac{1}{2}m_d r_d^2\right) + \frac{1}{2}m_a r_a^2 = m_d r_d^2 + \frac{1}{2}m_a r_a^2$$

6. How would increasing the mass of the disks or the axle affect the moment of inertia of the yo-yo?

If the mass of the disks or the axle are increased, the moment of inertia is also increased.

7. How would increasing the radius of the disks or the axle affect the moment of inertia of the yo-yo?

If the radii of the disks or the axle are increased, the moment of inertia is also increased.

8. The kinetic energy of the yo-yo has both a linear and a translational component:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \text{where} \quad \omega = \frac{v}{r_a}$$

Substitute the equation for ω in the equation for kinetic energy and simplify the result as much as possible, factoring out both $\frac{1}{2}$, v^2, and *m*.

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r_a}\right)^2 = \frac{1}{2}mv^2 + \frac{Iv^2}{2r_a^2} = \frac{1}{2}v^2\left(m + \frac{mI}{mr_a^2}\right) = \frac{1}{2}mv^2\left(1 + \frac{I}{mr_a^2}\right)$$

9. The potential energy of the yo-yo is:

$$U = mgy$$

Write the expression for the total mechanical energy *K* + *U* of the yo-yo in terms of *I*, *m*, *v*, r_a, *g*, and *y*.

$$E = K + U = \frac{1}{2}mv^2\left(1 + \frac{I}{mr_a^2}\right) + mgy$$

10. Substitute the total mass of the yo-yo $2m_d + m_a$ for *m* (not *I* yet) in the equation in step 9.

$$E = \frac{1}{2}(2m_d + m_a)v^2\left(1 + \frac{I}{(2m_d + m_a)r_a^2}\right) + (2m_d + m_a)gy$$

11. Substitute the equation for *I* in step 5 for *I* in step 10. This equation should now tell you nearly everything you need to know about the energy of your yo-yo.

$$E = \frac{1}{2}(2m_d + m_a)v^2\left(1 + \frac{m_d r_d^2 + \frac{1}{2}m_a r_a^2}{(2m_d + m_a)r_a^2}\right) + (2m_d + m_a)gy$$

12. Now **CAREFULLY** experiment with the yo-yo outside. Do **NOT** release the yo-yo from your finger while it is moving, drop it on the floor, knock it into anyone or anything, spin it around in circles, or do anything that would cause the string to break. What do you need to do to get the yo-yo to continue to return to your hand?

13. Why does the yo-yo finally stop if you don't pull on it?

The energy is dissipated through friction.

For more information see:
https://wiki.brown.edu/confluence/download/attachments/2752887/Yo-yo.pdf?version=1