Rotating Spheres

You will determine whether a solid sphere or a hollow sphere, both with the axes of rotation at the center, has a greater speed at the bottom of an incline.

You know the following:

$h_f = 0\ m$

$v_0 = 0\ m/s$

$\omega_0 = 0\ rad/s$

$h = h_0$

$$v_f = \sqrt{\frac{2mgh_0}{m + \frac{I}{r^2}}}$$

1) What is the equation for the moment of inertia for a solid sphere, with its axis through the center? Use *s* as the subscript for *m* and *r*.

$$I = \frac{2}{5} m_s r_s^2$$

2) Substitute this equation for *I* in the equation for v_f and simplify as much as possible.

$$v_f = \sqrt{\frac{2m_s gh_0}{m_s + \frac{\frac{2}{5} m_s r_s^2}{r_s^2}}}$$

$$v_f = \sqrt{\frac{2m_s gh_0}{m_s + \frac{2}{5} m_s}}$$

$$v_f = \sqrt{\frac{2m_s gh_0}{\frac{7}{5} m_s}}$$

$$v_f = \sqrt{\frac{10gh_0}{7}} = 1.20\sqrt{gh_0}$$

3) What is the equation for the moment of inertia for a hollow (thin-walled) sphere, with its axis through the center? Use *h* as the subscript for *m* and *r*.

$$I = \frac{2}{3} m_h r_h^2$$

4) Substitute this equation for I in the equation for v_f and simplify as much as possible.

$$v_f = \sqrt{\frac{2m_h g h_0}{m_h + \frac{\frac{2}{3}m_h r_h^2}{r_h^2}}}$$

$$v_f = \sqrt{\frac{2m_h g h_0}{m_h + \frac{2}{3}m_h}}$$

$$v_f = \sqrt{\frac{2m_h g h_0}{\frac{5}{3}m_h}}$$

$$v_f = \sqrt{\frac{6gh_0}{5}} = 1.10\sqrt{gh_0}$$

5) Which reaches the bottom of the incline first?

$$1.20\sqrt{gh_0} > 1.10\sqrt{gh_0}$$

The solid sphere has the greater translational speed than the hollow sphere and arrives at the bottom of the incline first.