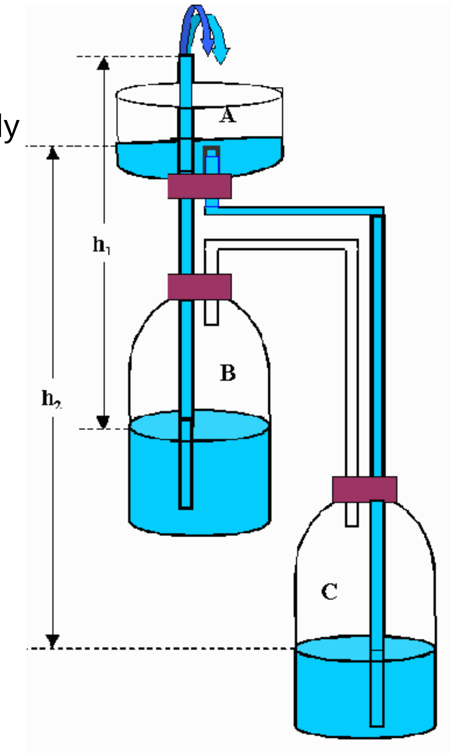
Hero's Fountain Activity

Equipment:

- three 2-liter plastic soda bottles
- three rubber stoppers with two holes in each
- two pieces of rigid plastic tubing, available from aquarium supply stores
- flexible plastic tubing, also available from aquarium supply stores



Components:

- bottle A with the fountain tube, in which water is poured to get the fountain started
- filled bottle B, into which air is forced from bottle C
- empty bottle C, into which water is forced from bottle A

Physics:

1. The water from bottle A flows into bottle C, which contains air, producing hydrostatic pressure $P_2 = \rho g h_2$, in addition to the initial atmospheric pressure P_{atm}, of the air in bottle C.
2. As a result, the pressure forces air up into bottle B, and the air transfers this pressure onto the water in bottle B.
3. The pressure in bottle B exerted by the column of water going up to bottle A is $P_1 = \rho g h_1$.
4. When the pressure in bottles B and C exceeds P_1 it forces the water to spout out of the fountain in bottle A.
5. Using Bernoulli's Principle we can determine the speed of the water rising out of the tube in bottle A:

$$P_{atm} + \rho g h_1 + \rho v^2/2 = P_{atm} + \rho g h_2 \quad \rightarrow \quad g h_1 + v^2/2 = g h_2 \quad \rightarrow$$

$$2gh_1 + v^2 = 2gh_2 \quad \rightarrow \quad v^2 = 2g(h_2 - h_1) \quad \rightarrow$$

$$v = [2g(h_2 - h_1)]^{1/2}$$

6. We can increase this speed by lowering bottle C, increasing the difference between h_2 and h_1, and decrease it by raising bottle C, decreasing the difference between h_2 and h_1.
7. We can calculate the height h_3 of the water spouting out of the fountain for a droplet of water with mass m using the Law of Conservation of Energy, where the potential energy is equal to the kinetic energy:

$$mgh_3 = mv^2/2 \quad \rightarrow \quad gh_3 = v^2/2 \quad \rightarrow$$

$$v^2 = 2gh_3 \quad \rightarrow \quad h_3 = v^2/2g \quad \rightarrow$$

$$h_3 = 2g(h_2 - h_1)/2g \quad \rightarrow \quad h_3 = h_2 - h_1$$

For additional information see:

http://www.rose-hulman.edu/~moloney/AppComp/2001Entries/e09k/fountain.htm