Building a Space Station

As the Planning Team of the Intergalactic Visitors Agency, you have been given the task of constructing one new space station for visitors from two different planets in distant solar systems. The space station will have three separate rings, one for Andromedians, one for Earthlings, and the other for Eridanians, each with the same rotation period.

The Intergalactic Visitors Agency translator has spoken with the leaders of both groups and she provides you with the following information about the two planets from which your visitors have traveled, and included information for Earth:

Planet	Planet mass M_p in kg	Planet radius R_p in m	Planet local gravity g_p in m/s^2	Space station ring radius r_p in m	Space station rotation period T in s	Space station ring linear velocity v_p in m/s
Andromedia	1.90E+27	71,492,000	24.8	3,796	77.7	307
Earth	5.97E+24	6,378,000	9.81	1,500	77.7	121.2
Eridania	1.31E+22	1,160,000	0.65	99.5	77.7	8.0

1. What is the equation for local gravity g_p in terms of G, the mass of the planet M_p, and the radius of the planet R_p?

$$F_p = \frac{GM_p m}{R_p^2} = mg_p \quad \rightarrow \quad g_p = \frac{GM_p}{R_p^2}$$

2. Calculate the local gravity for Andromedia and Eridania

$$g_{Andromedia} = \frac{(6.67\,x\,10^{-11}\,N\,m^2/kg^2)(1.90\,x\,10^{27}\,kg)}{(71{,}492{,}000\,m)^2} = 24.8\,m/s^2$$

$$g_{Eridania} = \frac{(6.67\,x\,10^{-11}\,N\,m^2/kg^2)(1.31\,x\,10^{22}\,kg)}{(1{,}160{,}000\,m)^2} = 0.65\,m/s^2$$

3. What is the equation for centripetal acceleration a_{cp} in terms of the space station ring radius r_p and the space station rotation period T? Remember that for each ring, $a_{cp} = g_p$.

$$a_{cp} = \frac{4\pi^2 r_p}{T^2}$$

4. Using this equation with $a_{cp} = g_{Earth}$ and $r_p = r_{Earth}$ solve the equation for T.

$$g_{Earth} = \frac{4\pi^2 r_{Earth}}{T^2} \quad \rightarrow \quad T^2 = \frac{4\pi^2 r_{Earth}}{g_{Earth}} \quad \rightarrow \quad T = 2\pi\sqrt{\frac{r_{Earth}}{g_{Earth}}}$$

5. Find the numeric value of the space station rotation period T. Remember that the space station rotation period T is the same for all three of the space station rings.

$$T = 2(3.14)\sqrt{\frac{1{,}500\ m}{9.81\ m/s^2}} = 77.7\ s$$

6. What is the equation for space station ring radius r_p in terms of local gravity g_p, and space station rotation period T? Remember that for each ring, $a_{cp} = g_p$.

$$a_{cp} = g_p = \frac{4\pi^2 r_p}{T^2} \quad \rightarrow \quad r_p = \frac{T^2 g_p}{4\pi^2}$$

7. Calculate the radii of the two space station rings in which your visitors will stay.

$$r_{Andromeda} = \frac{(77.7\ s)^2(24.8\ m/s^2)}{4(3.14)^2} = 3{,}796\ m$$

$$r_{Eridania} = \frac{(77.7\ s)^2(0.65\ m/s^2)}{4(3.14)^2} = 99.5\ m$$

8. What is the equation for centripetal acceleration a_{cp} in terms of a space station ring radius r_p and space station ring linear velocity v_p?

$$a_{cp} = \frac{v_p^2}{r_p}$$

9. Solve the equation above for v_p

$$v_p^2 = a_{cp} r_p \quad \rightarrow \quad v_p = \sqrt{a_{cp} r_p}$$

10. Calculate the numeric value of the linear velocity of each ring. Remember that $a_{cp} = g_p$ and that the linear velocities will not be the same.

$$v_{Andromedia} = \sqrt{(24.8\ m/s^2)(3{,}796\ m)} = 307\ m/s$$

$$v_{Earth} = \sqrt{(9.81\ m/s^2)(1{,}500\ m)} = 121\ m/s$$

$$v_{Eridania} = \sqrt{(0.65\ m/s^2)(99.5\ m)} = 8.0\ m/s$$