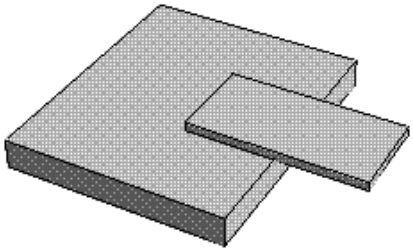
Balancing the Books

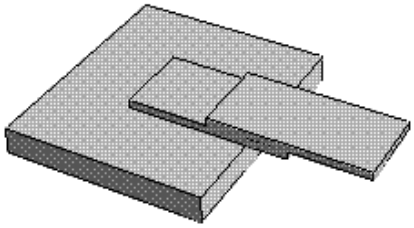
You will need to work as a team during this activity, and you will likely be more successful if you complete the calculations first. Be sure that you read **all** of the directions before you begin.

1. In this activity you will be stacking books or other sets of identical objects on top of each other in a stack, extending the stack as far as possible over the edge of the desk, creating the maximum possible overhang. You may adjust the stack as needed, however, no external supports are allowed, and the stack must balance on its own.

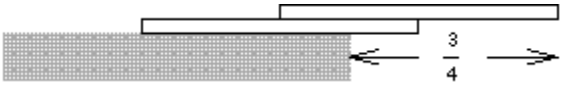


one book/object

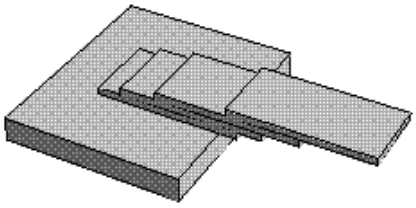
maximum overhang is ½ the book/object length



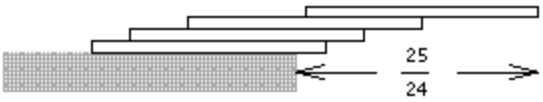
two books/objects



maximum overhang is ¾ the book/object length



four books/objects



maximum overhang is 25/24 the book/object length

Images: http://mathforum.org/advanced/robertd/harmonic.html

2. We could develop a general formula for this activity, but that would take a long time. Instead we can use some of the solutions that already exist, http://mathforum.org/advanced/robertd/harmonic.html and http://www.brpreiss.com/books/opus5/html/page46.html. The first website provides an explanation of harmonic numbers. The series $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ is called the harmonic series and $H_n = \sum_{k=1}^{n} \frac{1}{k}$ generates the harmonic numbers. For example:

$$H_1 = \sum_{k=1}^{1} \frac{1}{k} = 1 \qquad H_2 = \sum_{k=1}^{2} \frac{1}{k} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$H_3 = \sum_{k=1}^{3} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6} \qquad H_4 = \sum_{k=1}^{4} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$$

The second website provides a derivation for γ (gamma) = 0.5772, first discovered by mathematician Leonard Euler, and used to calculate some of the numbers in the table below.

3. Record the length *L* of your text book or other object in either inches or centimeters, as accurately as possible: _____

4. Complete the following table. Write all numbers with 4 decimal places. Be sure to specify the units you used in columns D and F.

Book/Object Stacking Data

A	B = 1/A	C = sum(B)	D = $\frac{C}{2} x\ L$	E = ln(A)	F = $\left(\frac{E+\gamma}{2}\right) x\ L$	G = $\left(\frac{D-F}{F}\right) x 100$
Number of books/ objects *n*	Harmonic series 1/*n*	Harmonic number $H_n = \sum_{k=1}^{n} \frac{1}{k}$	Experimental maximum overhang in _____	ln(*n*)	Calculated maximum overhang in _____	Percent difference between experimental and calculated maximum overhang
1	1.0000	1.0000		0.0000		
2	0.5000	1.5000		0.6931		
3						
4						
5						
6						
7						
8						
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14						
15						

5. Using a piece of **graph paper neatly** plot by hand Experimental maximum overhang versus Number of books. **On the same graph** plot Calculated maximum overhang versus Number of books. Be sure to **include titles for the graph, both axes, units, and clearly indicate which line is the experimental data and which is the calculated data.**

6. Note that at about 15 books/objects, the percent difference between the experimental and calculated results should be as little as about 1%. If we were to continue to add books/objects do you think that percent difference would increase or decrease? Explain your reasoning.

7. What is the calculated maximum overhang for a stack of 100 of the books/objects you used? Show your work.

8. Now you may begin stacking books/objects. Measure the maximum overhang you and your team were able to achieve with the books/objects that you used: _____. Be sure to include the units, which must be the same as the units used to measure the books/objects in step 3.

9. How well did your book/object stacking efforts agree with your calculations? Explain.

10. How does this activity demonstrate the concept of center of mass and/or center of gravity? Be specific.