Problem: Jack and Jill are working on a project together. Jack can complete the project by himself in 6 hours, which means he can complete $\frac{1}{6}$ of the project per hour. Jill can complete the project by herself in 5 hours, which means she can complete $\frac{1}{5}$ of the project per hour.

Known: The amount of the project each individual can complete in 1 hour working **alone**.

Unknown: The amount of time it takes for them to complete the project working **together**.

Find: How long does it take them to complete the project if they work together?

Setup: (amount Jack does per hour + amount Jill does per hour)(number of hours) = completed project. Let *t* be the number of hours it takes for them to complete the project working together.

Solve: $$\left(\frac{1}{6}+\frac{1}{5}\right)t=1$$

$$\frac{t}{6}+\frac{t}{5}=1$$

multiply both sides by the common denominator, 30:

$$5t+6t=30$$

$$11t=30$$

$$t=\frac{30}{11}=2\frac{8}{11}=2.727\ hrs=2\ hrs,\ 43.6\text{ min working together}$$

Check: $$\left(\frac{1}{6}+\frac{1}{5}\right)\left(\frac{30}{11}\right)=1$$

$$\left(\frac{5}{30}+\frac{6}{30}\right)\left(\frac{30}{11}\right)=1$$

$$\left(\frac{11}{30}\right)\left(\frac{30}{11}\right)=1$$

Problem: Jack and Jill want to paint their living room. Jack can paint the living room alone in 5 hours. Together he and Jill can paint the living room in 2 hours.

Known: The amount of time it takes them to paint the living room if they do it together and the amount of time it takes Jack working alone.

Unknown: The amount of time it takes Jill to paint the living room working **alone**.

Find: How long would it take Jill to paint the living room alone?

Setup: Jack can paint $\frac{1}{5}$ of the living room per hour. Let *t* be the number of hours it would take Jill to paint the living room working alone. She can paint $\frac{1}{t}$ of the living room per hour. Since Jack and Jill can complete the entire job in 2 hours, the sum of the amounts done by each must be equal to 1.

In *2* hours Jill completes $2\left(\frac{1}{t}\right) = \frac{2}{t}$ of the living room.

In *2* hours Jack completes $2\left(\frac{1}{5}\right) = \frac{2}{5}$ of the living room

Solve: $\frac{2}{5} + \frac{2}{t} = 1$

multiply both sides by the common denominator, *5t*:

$2t + 10 = 5t$

$3t = 10$

$t = \frac{10}{3} = 3\frac{1}{3} = 3.33\ hrs = 3\ hrs,\ 20\ \text{min}$ for Jill to paint the living room working alone

Check:

$\frac{2}{5} + \frac{2}{\frac{10}{3}} = 1$

$\frac{2}{5} + 2\left(\frac{3}{10}\right) = 1$

$\frac{2}{5} + \frac{6}{10} = 1$

$\frac{4}{10} + \frac{6}{10} = 1$

Problem: Ben and Jerry are each working on their own on the *New York Times* crossword puzzle. It takes Jerry, working alone, 3 hours longer than Ben to finish the puzzle. If they work on the puzzle together it takes 2 hours.

Known: The amount of time it takes them to finish the crossword puzzle if they do it **together**.

Unknown: The amount of time it takes each of them to complete the crossword puzzle working **alone**.

Find: How long would it take each of them, working alone, to complete the puzzle?

Setup: t is the time it takes Ben to finish the puzzle working alone.

In *2* hours Ben completes $\frac{2}{t}$ of the crossword puzzle.

In *2* hours Jerry completes $\frac{2}{t+3}$ of the crossword puzzle.

Solve: $\frac{2}{t}+\frac{2}{t+3}=1$

multiply both sides by the common denominator, $(t)(t+3)$:

$$(t)(t+3)\left(\frac{2}{t}+\frac{2}{t+3}\right)=(t)(t+3)(1)$$

$$2(t+3)+2t=t^2+3t$$

$$2t+6+2t=t^2+3t$$

$$4t+6=t^2+3t$$

$$t^2-t-6=0$$

$$(t-3)(t+2)=0$$

$t=3\ hrs$ for Ben to complete the crossword puzzle working alone

$t=3+3=6\ hrs$ for Jerry to complete the crossword puzzle working alone

Check: $\frac{2}{3}+\frac{2}{3+3}=1$ $\qquad \frac{4}{6}+\frac{2}{6}=\frac{6}{6}=1$

Problem: Bill and Hillary are typing a term paper. It takes Bill twice as long to type the same amount as Hillary. Working together it takes them 5 hours to type the entire term paper.

Known: The amount of time it takes them to type the entire term paper working **together** and the relative rate.

Unknown: The amount of time it takes each of them to type the entire term paper working **alone**.

Find: How long would it take each of them to type the entire term paper working alone?

Setup: Hillary can type $\frac{1}{t}$ of the term paper per hour. In 5 hours she can type $\frac{5}{t}$ of the term paper.

Bill can type of the term paper per hour, since it takes Bill *2t* as long as Hillary. In 5 hours he can type $\frac{1}{2t}$ of the term paper.

Solve: $$\frac{5}{t}+\frac{5}{2t}=1$$

multiply both sides by the common denominator, *2t:*

$$2t\left(\frac{5}{t}+\frac{5}{2t}\right)=2t(1)$$

$$10+5=2t$$

$$15=2t$$

$$t=\frac{15}{2}=7.5\ hrs=7\ hrs,\ 30\text{ min}$$

$t=7\ hrs$, 30 min for Hillary working alone,

$2t=15\ hrs$ for Bill working alone

Check: $$\frac{5}{\frac{15}{2}}+\frac{5}{2\left(\frac{15}{2}\right)}=1 \qquad 5\left(\frac{2}{15}\right)+\frac{5}{2}\left(\frac{2}{15}\right)=1$$

$$\frac{10}{15}+\frac{5}{15}=\frac{15}{15}=1$$

Problem: Jack and Jill agreed to meet their friends Larry and Lori in Las Vegas. Jack and Jill traveled 250 miles by train and Larry and Lori traveled 300 miles by train. The train that Larry and Lori were on traveled 20 mph faster than the train that Jack and Jill were on, but each spent the same amount of time traveling. How fast were they traveling? How long did the trip take?

Known:

	distance in miles	rate in miles/hr	time in hrs
Jack and Jill	250	r	$t = \frac{250}{r}$
Larry and Lori	300	r + 20	$t = \frac{300}{r+20}$

distance = rate *x* time time = distance/rate

times are the same

Unknown: rates and time

Find: How fast did Jack and Jill travel? How fast did Larry and Lori travel? How long had they been traveling?

Setup: $\frac{250}{r} = \frac{300}{r+20}$

Solve: multiply both sides by the common denominator, $(r)(r+20)$:

$$(r)(r+20)\left(\frac{250}{r}\right) = (r)(r+20)\left(\frac{300}{r+20}\right)$$

$(r+20)(250) = (r)(300)$ $250r + 5{,}000 = 300r$

$5{,}000 = 50r$ $r = \frac{5{,}000}{50} = 100\ miles/hr$

$r = 100\ miles/hr$, Jack and Jill's rate

$r + 20 = 120\ miles/hr$, Larry and Lori's rate

$t = \frac{250}{100} = 2.5\ hrs$ travel time

Check: $\frac{250}{100} = \frac{300}{100+20}$ $\frac{250}{100} = \frac{300}{120}$

Problem: Lee and Lana enter a race, which can be completed by foot or by bicycle. Lee decides to walk. Lana, however, decides to race on her bicycle and her speed is 13 mph faster than Lee's speed walking. In the time that Lana rode 16 miles Lee walked 4 miles.

Known:

	distance in miles	rate in miles/hr	time in hrs
Lee	4	r	$t = \frac{4}{r}$
Lana	16	r + 13	$t = \frac{16}{r+13}$

distance = rate *x* time time = distance/rate

times are the same

Unknown: rates and time

Find: How fast did Lee walk? How fast did Lana ride? How long had they been walking and riding?

Set up: $\frac{4}{r} = \frac{16}{r+13}$

Solve: multiply both side by the common denominator, $(r)(r+13)$:

$$(r)(r+13)\left(\frac{4}{r}\right) = (r)(r+13)\left(\frac{16}{r+13}\right)$$

$(r+13)(4) = (r)(16)$ $4r + 52 = 16r$

$12r = 52$

$r = \frac{52}{12} = \frac{13}{3} = 4\frac{1}{3} = 4.33\ miles/hr$ for Lee

$r = 4\frac{1}{3} + 13 = 17\frac{1}{3} = 17.33\ miles/hr$ for Lana

Check: $\frac{4}{\frac{13}{3}} = \frac{16}{17\frac{1}{3}}$ $\frac{4}{\frac{13}{3}} = \frac{16}{\frac{52}{3}}$ $\frac{12}{13} = \frac{48}{52}$

Problem: Emily inherits $100,000 and invests it in two certificates of deposit. One pays 6% and the other pays 4.5% simple interest annually. She earns a total of $5,025 per year in interest.

Known:

	6% account	4.5% account	total
amount invested	x	y	100,000
interest rate	0.06	0.045	
total interest	$0.06x$	$0.045y$	5,025

Unknown: amounts invested in each account

Find: How much did she invest in each account?

Setup: $x + y = 100{,}000$

$$0.06x + 0.045y = 5{,}025$$

Solve: $y = 100{,}000 - x$

$$0.06x + 0.045(100{,}000 - x) = 5{,}025$$

$$0.06x + 4{,}500 - 0.045x = 5{,}025$$

$$0.015x + 4{,}500 = 5{,}025$$

$$0.015x = 525$$

$x = \frac{525}{0.015} = \$35{,}000$ in the 6% account

$$y = 100{,}000 - 35{,}000 = \$65{,}000$$

Check: $35,000 + $65,000 = $100,000

$$0.06(35{,}000) + 0.045(65{,}000) = 2{,}100 + 2{,}925 = \$5{,}025$$

Problem: Vanilla jelly beans cost $1.35 a pound and blueberry jelly beans cost $1.95 a pound. They are mixed to create a 30 pound mixture that costs $1.70 a pound.

Known:

jelly beans	vanilla	blueberry	total
amount	x	y	30
cost per pound	1.35	1.95	
total	1.35x	1.95y	1.70(30)

Unknown: amounts of each type of jelly bean

Find: How many pounds of vanilla and blueberry jelly beans were needed?

Setup:

$$x + y = 30$$

$$1.35x + 1.95y = 1.70(30)$$

Solve:

$$x = 30 - y$$

$$1.35(30 - y) + 1.95y = 1.70(30)$$

$$1.35(30 - y) + 1.95y = 51.0$$

$$40.5 - 1.35y + 1.95y = 51.0$$

$$40.5 + 0.60y = 51.0$$

$$0.60y = 10.5$$

$$y = \frac{10.5}{0.60} = 17.5 \text{ } \textit{pounds} \text{ of blueberry jelly beans}$$

$$x = 30 - 17.5 = 12.5 \text{ } \textit{pounds} \text{ of vanilla jelly beans}$$

Check:

$$12.5 + 17.5 = 30$$

$$1.35(12.5) + 1.95(17.5) = 1.70(30)$$

$$16.875 + 34.125 = 51$$

Problem: Three roommates, Angela, Bonnie, and Celina, decide to build some bookshelves. Together they can build them in 3 hours. Working alone, Angela could build the bookshelves in 8 hours. Working alone, Bonnie would take 10 hours to complete the bookshelves.

Known: The amount of time it takes them to make the bookshelves if they do it together and the amount of time it takes Angela and Bonnie if they do it alone.

Unknown: The amount of time it takes Celina to make the bookshelves working **alone**.

Find: How long would it take Celina to make the bookshelves working alone?

Setup: Angela can build $\frac{1}{8}$ of the bookshelves per hour working alone.

Bonnie can build $\frac{1}{10}$ of the bookshelves per hour working alone.

Celina can build $\frac{1}{t}$ of the bookshelves per hour working alone.

Solve: $3\left(\frac{1}{8}+\frac{1}{10}+\frac{1}{t}\right)=1$

multiply both sides by the common denominator, *40t*:

$(40t)(3)\left(\frac{1}{8}+\frac{1}{10}+\frac{1}{t}\right)=(40t)(1)$

$15t+12t+120=40t$

$27t+120=40t$

$13t=120$

$t=\frac{120}{13}=9\frac{3}{13}=9.23\ days$ for Celina working alone to build the bookshelves

Check: $3\left(\frac{1}{8}+\frac{1}{10}+\frac{1}{\frac{120}{13}}\right)=1$ $3\left(\frac{1}{8}+\frac{1}{10}+\frac{13}{120}\right)=1$

$3\left(\frac{15}{120}+\frac{12}{120}+\frac{13}{120}\right)=1$ $3\left(\frac{40}{120}\right)=\frac{120}{120}=1$

Problem: To reach an appointment 50 miles away, Emily allowed 1 hour. After driving 30 miles, she realized that her speed would have to be increased 15 miles per hour for the remainder of the trip.

Known:

	distance in miles	rate in miles/hr	time in hrs
First part of trip at lower rate	30	r	$t_1 = \frac{30}{r}$
Second part of trip at higher rate	20	r + 15	$t_2 = \frac{20}{r+15}$
Total	50		$t_1 + t_2 = 1$

distance = rate *x* time time = distance/rate

Unknown: rates of first and second parts of the trip

Find: What were Emily's rates for the first and second parts of her trip?

Setup: $\frac{30}{r} + \frac{20}{r+15} = 1$

Solve: multiply both sides by the common denominator, $(r)(r+15)$:

$$(r)(r+15)\left(\frac{30}{r} + \frac{20}{r+15}\right) = (r)(r+15)(1)$$

$$30(r+15) + 20(r) = (r)(r+15)$$

$$30r + 450 + 20r = r^2 + 15r$$

$$50r + 450 = r^2 + 15r$$

$$r^2 - 35r - 450 = 0$$

$$(r-45)(r+10) = 0$$

$r - 45 = 0$ or $r + 10 = 0$

$r = 45\ miles/hr$ for the first part of the trip (r can't be -10)

$r + 15 = 45 + 15 = 60\ miles/hr$ for the second part of the trip

Check: $\frac{30}{45} + \frac{20}{45+15} = 1$ $\frac{30}{45} + \frac{20}{60} = 1$ $\frac{120}{180} + \frac{60}{180} = 1$