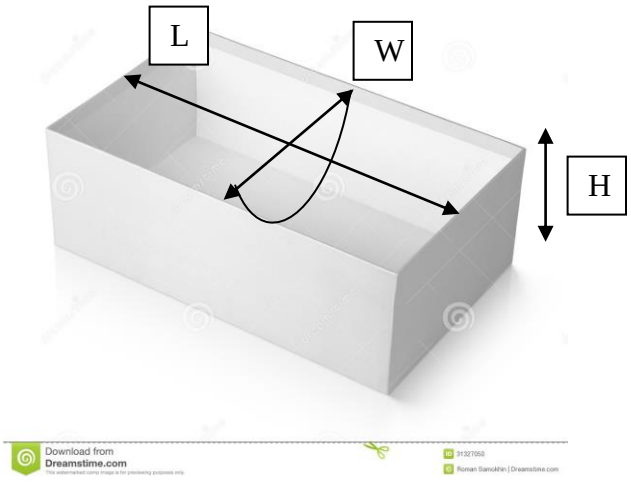
Shoe Box Oven Activity

We'll figure out if it is possible to cook a marshmallow in a shoe box lined with aluminum foil.



1. W = width of shoe box
2. H = height of shoe box
3. L = length of shoe box
4. the origin (0, 0), is located at the intersection of the L and W axes
5. the marshmallow is located on a skewer along the L axis, at (0, 0)
6. the marshmallow is a distance of $\frac{1}{2}$W from the left side of the shoe box and a distance $\frac{1}{2}$W from the right side of the shoebox, a distance $\frac{1}{2}$L from the top of the shoe box and a distance $\frac{1}{2}$L from the bottom of the shoe box
7. assume that the inside of the shoe box, along axis L, is covered in foil, shaped in a parabola, with the equation $ax^2+bx+c=y$
8. data:

point	equation	equation	simplified
$(x,y)=\left(\frac{-1}{2}W,0\right)$ left side of box	(1)	$a\left(\frac{-1}{2}W\right)^2+b\left(\frac{-1}{2}W\right)+c=0$	$\frac{1}{4}aW^2-\frac{1}{2}b+c=0$
$(x,y)=\left(\frac{1}{2}W,0\right)$ right side of box	(2)	$a\left(\frac{1}{2}W\right)^2+b\left(\frac{1}{2}W\right)+c=0$	$\frac{1}{4}aW^2+\frac{1}{2}b+c=0$
$(x,y)=(0,-H)$ bottom center of box	(3)	$a(0)^2+b(0)+c=-H$	$c=-H$

9. $\frac{1}{4}aW^2-\frac{1}{2}bW-H=0$ eq. (1), substitute $c=-H$
10. $\frac{1}{4}aW^2+\frac{1}{2}bW-H=0$ eq. (2), substitute $c=-H$

11. $\frac{1}{2}aW^2 - 2H = 0$ eq. (1) + eq. (2)

12. $aW^2 - 4H = 0$ simplify

13. $aW^2 = 4H$ solve

14. $a = \frac{4H}{W^2}$

15. $\left(\frac{4H}{W^2}\right)\left(\frac{-1}{2}W\right)^2 + b\left(\frac{-1}{2}W\right) - H = 0$ eq. (1)

16. $\left(\frac{4H}{W^2}\right)\left(\frac{1}{2}W\right)^2 + b\left(\frac{1}{2}W\right) - H = 0$ eq. (2)

17. $-\left(\frac{4H}{W^2}\right)\left(\frac{1}{2}W\right)^2 - b\left(\frac{1}{2}W\right) + H = 0$ eq. (4) = eq. (2) x -1

18. $-2b\left(\frac{1}{2}W\right) = 0$ eq. (1) + eq. (4)

19. $b = 0$

20. $\left(\frac{4H}{W^2}\right)x^2 - H = y$ substitute *a*, *b*, and *c* in

21. $\left(\frac{4H}{W^2}\right)x^2 = y + H$

22. $x^2 = \left(\frac{W^2}{4H}\right)(y + H)$

23. $4p = \frac{W^2}{4H}$ *p* = y-coordinate of the focus

24. $p = \frac{W^2}{16H}$

25. $y = \left(\frac{4H}{W^2}\right)x^2 - H$

26. $\frac{dy}{dx} = \left(\frac{8H}{W^2}\right)x$

27. $\left(\frac{dy}{dx}\right)^2 = \left[\left(\frac{8H}{W^2}\right)x\right]^2 = \frac{64H^2}{W^4}x^2$

28. arc length = $\frac{1}{2}\sqrt{16H^2 + W^2} + \left(\frac{W^2}{8H}\right)\left[\ln(4H + \sqrt{16H + W^2}) - \ln(W)\right]$

(source: http://www.had2know.com/academics/parabola-segment-arc-length-area.html)

29. Access the Shoe Box Oven Activity Calculator and enter your own numbers for the width and length of the shoe box and the width and height of the marshmallow.

30. Given the numbers you selected, does your Shoe Box Oven have enough power to cook the marshmallow? Explain why or why not.