Geostationary Satellite

We know the following:

$M_E = 5.98\,x\,10^{24}\ kg$ $\qquad$ $r_E = 6.38\,x\,10^{6}\ m$ $\qquad$ $G = 6.67\,x\,10^{-11}\ N\ m^2\ /\ kg^2$

1) What is a geostationary satellite?

A geostationary satellite orbits the Earth once per day on a circular path that lies in the plane of the equator.

2) What is the period *T* in *s* of a geostationary satellite?

$$(1\ day)\left(\frac{24\ hr}{1\ day}\right)\left(\frac{60\ \text{min}}{1\ hr}\right)\left(\frac{60\ s}{1\ \text{min}}\right) = 86{,}400\ s$$

3) What is the equation for the distance r from the center of the Earth to the altitude of a geostationary satellite in terms of *G*, *T*, and M_E?

$$r^{3/2} = \frac{T\sqrt{GM_E}}{2\pi} \rightarrow (r^{3/2})^{2/3} = \left(\frac{T\sqrt{GM_E}}{2\pi}\right)^{2/3} \rightarrow r = \left(\frac{T\sqrt{GM_E}}{2\pi}\right)^{2/3}$$

4) What is the numerical value of the distance *r* from the center of the Earth to the altitude of a geostationary satellite?

$$r = \left(\frac{(86{,}400\ s)\sqrt{(6.67\ x\,10^{-11}\ N\ m^2\ /\ kg^2)(5.98\ x\,10^{24}\ kg)}}{2(3.14)}\right)^{2/3} = 4.23\,x\,10^{7}\ m$$

5) What is the height *H* of the satellite above the surface of the Earth?

$$H = r - r_E = 4.27\,x\,10^{7}\ m - 6.38\,x\,10^{6}\ m = 3.59\,x\,10^{7}\ m$$

6) Assume that the satellite travels in a circular orbit. What is the circumference of the orbit? Hint: this is the actual distance traveled by the satellite in one day.

$$C = 2\pi r = 2(3.14)(4.23\,x\,10^{7}\ m) = 2.656\,x\,10^{8}\ m$$

7) What is the linear orbital velocity *v* in m/s?

$$v = \frac{C}{t} = \frac{2.656\,x\,10^{8}\ m}{86{,}400\ s} = 3{,}074\ m/s$$

8) What is the angular orbital velocity ω in *rad/s*? Hint: how many degrees and radians does this satellite travel in one day?

$$\omega = \frac{\theta}{t} = \frac{360^{0}}{86{,}400\ s} = \frac{2\pi\ rad}{86{,}400\ s} = \frac{2(3.14)\ rad}{86{,}400\ s} = 7.269\,x\,10^{-5}\ rad/s$$